Lesmahagow High School

Mathematics Department

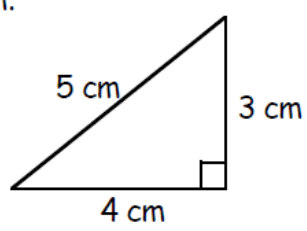
S2

Pythagoras

Pythagoras Theorem

Pythagoras was a famous Greek Mathematician who discovered an amazing connection between the three sides of a **right angled triangle**. This connection means it is possible to **CALCULATE** the length of one side of a right angle triangle as long as you know the lengths of the other two.

Look at this right angled triangle with sides 3cm, 4cm and 5cm.



If you add the two smaller sides (3cm and 4cm) together do you get the longer side (5cm) ? - NO.

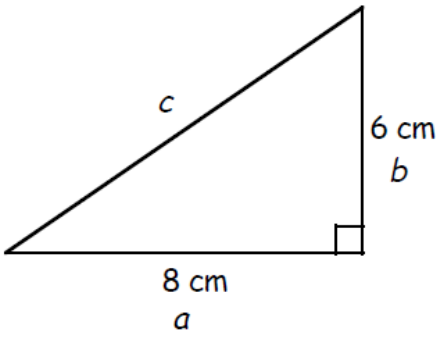
Can you see that $3^2 = 9$, $4^2 = 16$ and $5^2 = 25$?

Can you also see that:- $3^2 + 4^2 = 9 + 16 = 25 = 5^2$?

Pythagoras found that this connection between the three sides of a right angled triangle was true for every right angled triangle.

Example 1 :- The two smaller sides of this right angled triangle are 8 centimetres and 6 centimetres.

To calculate the length of the hypotenuse, use **Pythagoras' Rule**.



=> $c^2 = a^2 + b^2$

=> $c^2 = 8^2 + 6^2$

=> $c^2 = 64 + 36 = 100$

=> $c = \sqrt{100} = 10$ cm

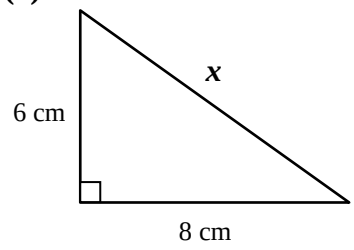
This is how you set down the working

Applying Geometric Skills to Sides and Angles of a Shape

Using Pythagoras' Theorem

1. Find the length of the hypotenuse, marked ***x***, in each of the following triangles.

(a)



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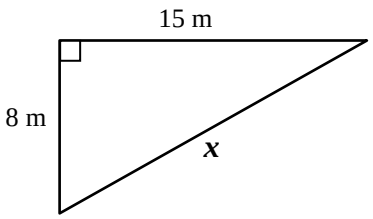

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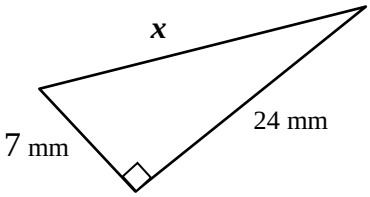

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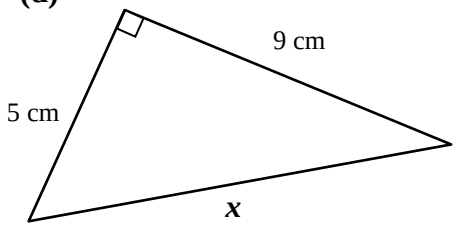

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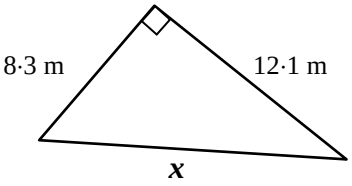

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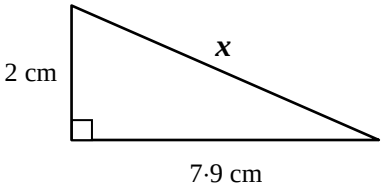

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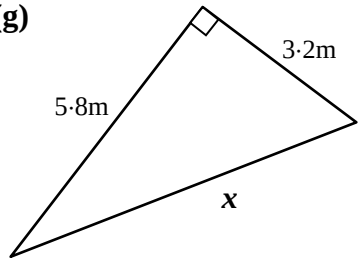

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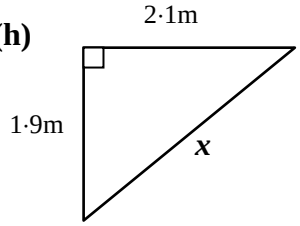
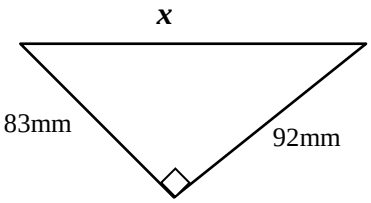


(i)



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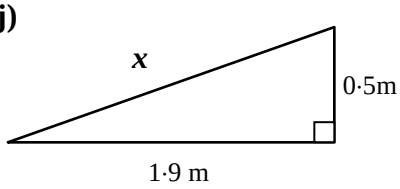

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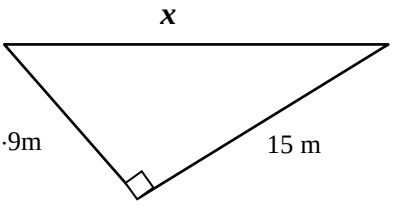

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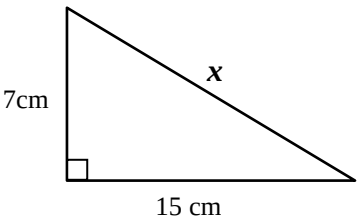

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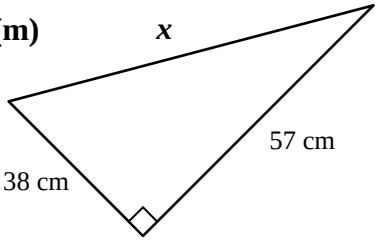

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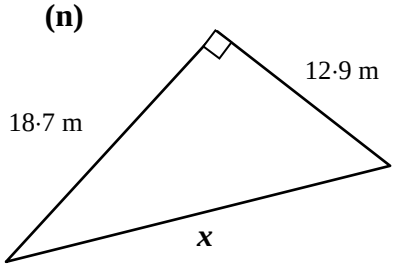

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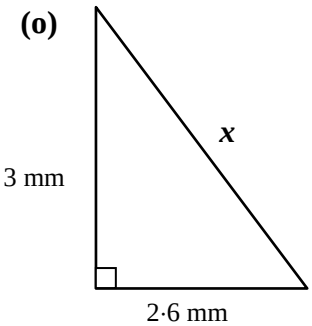


You can use Pythagoras' Rule to calculate one of the smaller sides as follows :-

Can you see this time that to find a smaller side (a) :-

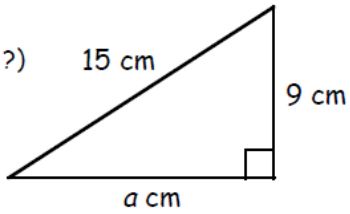
$$\Rightarrow \quad a^2 = c^2 - b^2$$

$$\Rightarrow \quad a^2 = 15^2 - 9^2$$

$$\Rightarrow \quad a^2 = 225 - 81 = 144$$

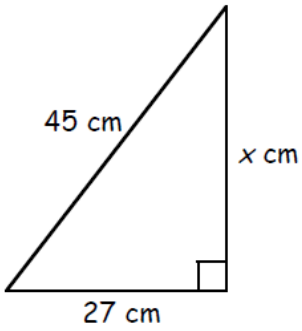
$$\Rightarrow \quad a = \sqrt{144} = 12 \text{ cm}$$

(can you see why the "-" sign ?)



Exercise 5

1. Calculate the length of the side of this right angled triangle marked with an x.



$$a^2 = c^2 - b^2$$

$$\Rightarrow \quad x^2 = 45^2 - 27^2$$

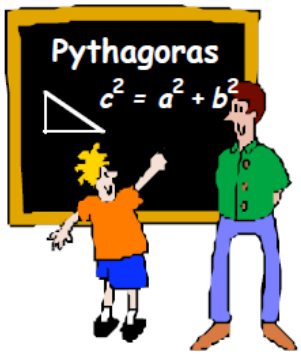
$$\Rightarrow \quad x^2 = 2025 - 729$$

$$\Rightarrow \quad x^2 = 1296$$

$$\Rightarrow \quad x = \ ? \text{ cm}$$

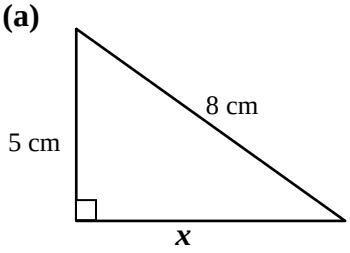
Problems involving Pythagoras Theorem

Whenever you come across a problem involving finding a missing side in a right angled triangle you should consider using Pythagoras' Rule to calculate its length



2. Find the length of the side, marked ***x***, in each of the following triangles.

(a)



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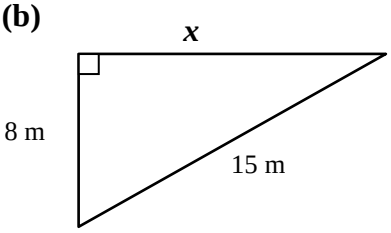

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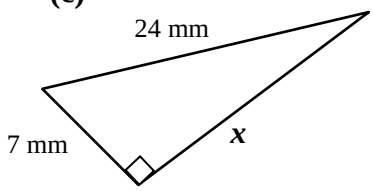

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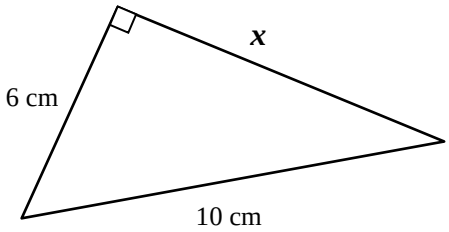

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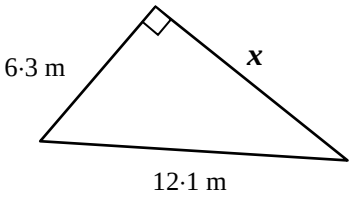

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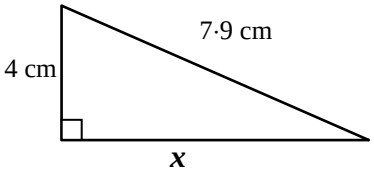

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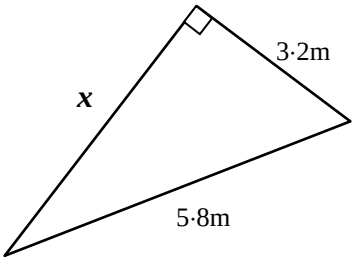

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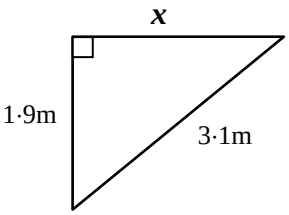

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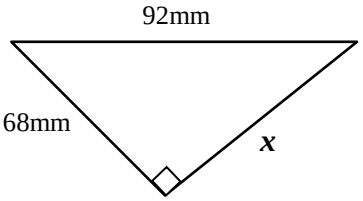

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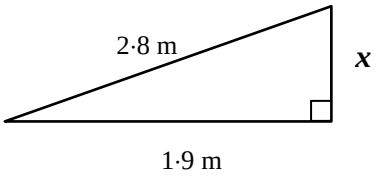

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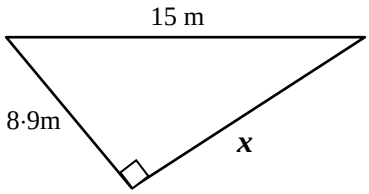

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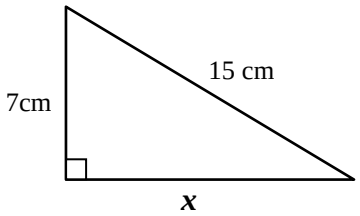

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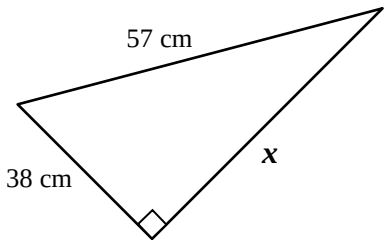

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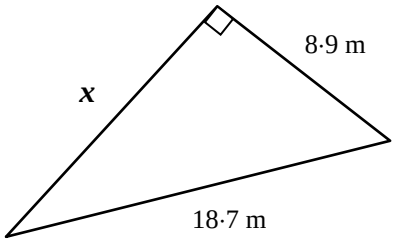

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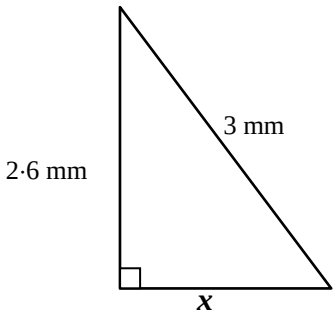
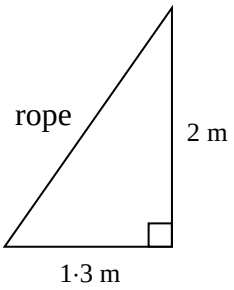


3. Guy ropes are used to support a tent pole.
The pole is 2 metres high and the guy rope
is fixed 1·3 metres from the bottom of the pole.

What is the length of the guy rope?



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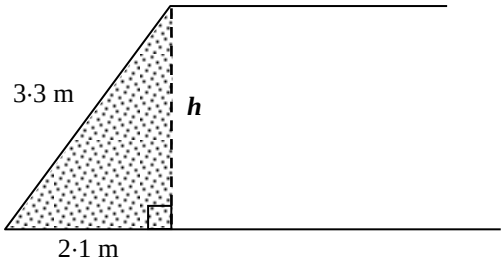
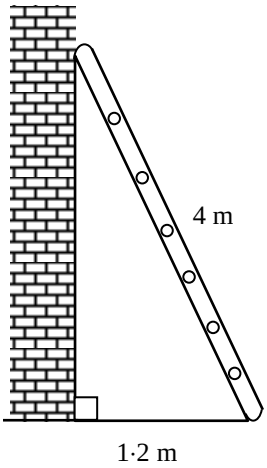


Jim's house has an attic room with a sloping end wall. He is going to make a fitted cupboard. What will be the height of the cupboard, ***h*** ?

5.

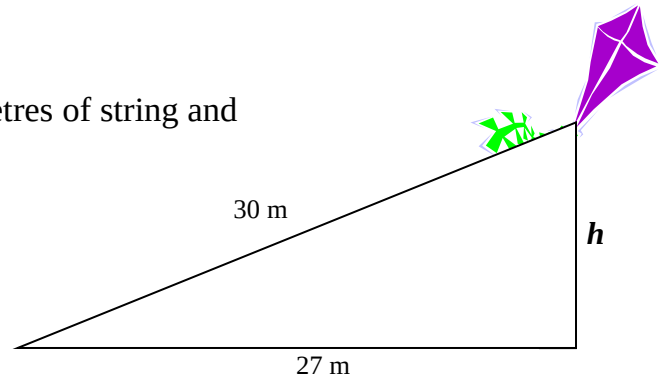


John' ladder is 4 metres long. He sets it up so that the foot of the ladder is 1·2 metres from the wall.

How far up the wall will the ladder reach?

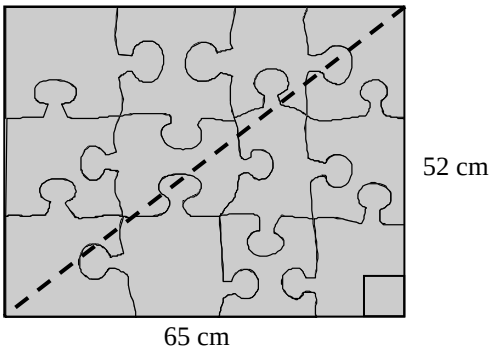
6. Eddie is flying his kite. He lets out 30 metres of string and moves 27 metres from his starting point.

How high is the kite above the ground?

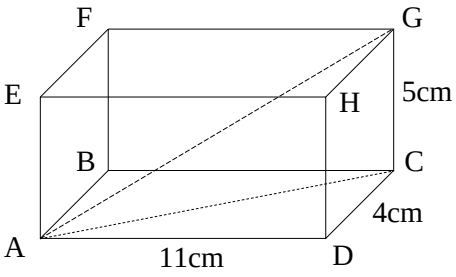


7. A rectangular jigsaw measures 65 cm by 52 cm.

What length is its diagonal?



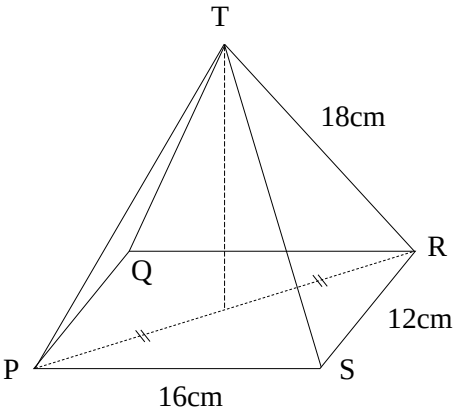
8. Consider the cuboid opposite.



(a) Calculate the length of the face diagonal AC.

(b) Hence calculate the length of the space diagonal AG.

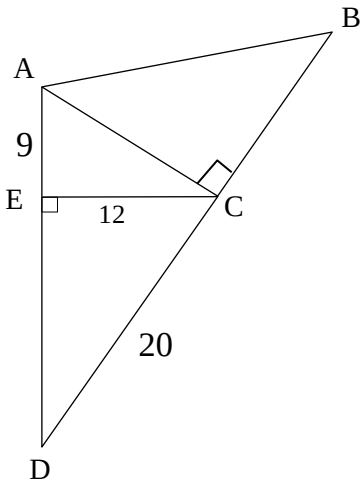
9. The pyramid opposite has a rectangular base.



(a) Calculate the length of the base diagonal PR.

(b) Given that edge TR = 18cm, calculate the vertical height of the pyramid.

10. Consider the diagram opposite. All lengths are centimetres.



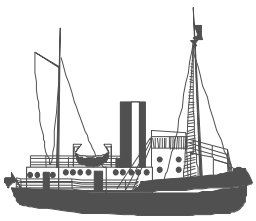
(a) Calculate the length of AC.

(b) Calculate the length of ED.

Draw a diagram to help you answer question 11 – 18

11. A ship sails 9km due North and then a further 17km due East.

How far is the ship from its starting point?



12. An aircraft flies 400km due West and then a further 150km due South.

How far is the aircraft from its starting point?

13. A ship sailed $8 \cdot 42$ km due East followed by $4 \cdot 7$ km due South.

How far would it have sailed if it had followed a direct course?

14. A ship sails 9km due North and then a further distance x km due West.

The ship is now 12km from its starting point. Calculate x.

15. How long is the diagonal of a square of side 11mm?

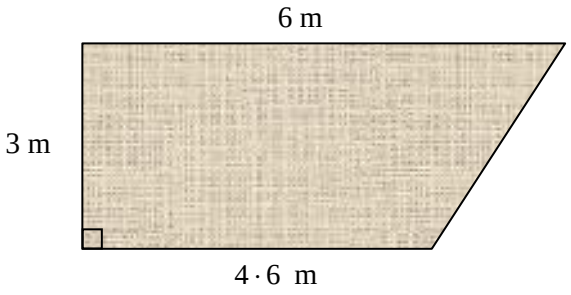
16. A rectangle measures 14cm by 9cm. Calculate the length of its diagonals.

17. A ladder of length 5 metres leans against a vertical wall with the foot of the ladder 2 metres from the base of the wall. How high up the wall does the ladder reach?

18. A ladder is placed against a vertical wall. If the distance between the foot of the ladder and the wall is $1 \cdot 8$ metres, and the ladder reaches 4 metres up the wall, calculate the length of the ladder.

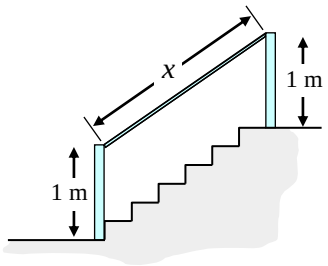
19. The room shown opposite has two parallel sides.

Using the given dimensions calculate the perimeter of the room.



20. Calculate the length of the banister rail shown in the diagram if there are 6 stairs, and if each tread measures 25cm and each riser 20cm.

Give your answer in metres.

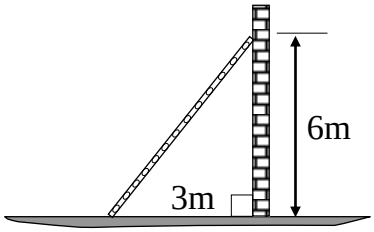


Pythagoras' Theorem (more practice)

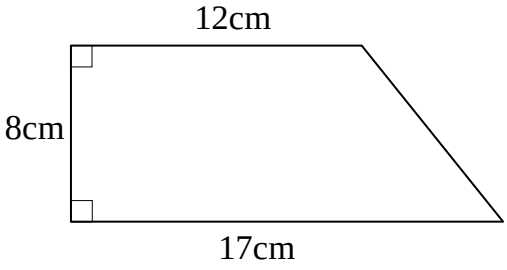
Answers should be rounded to 1-decimal place where necessary.

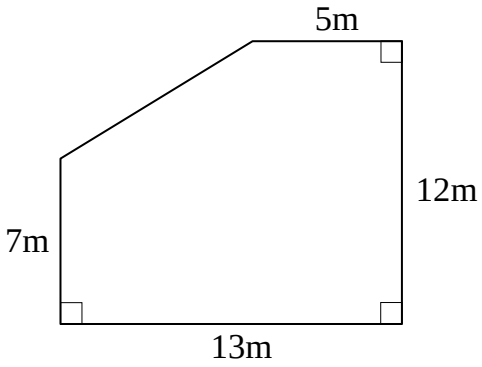
1. A ladder leans against a wall as shown in the diagram opposite

From the information given calculate the length of the ladder.

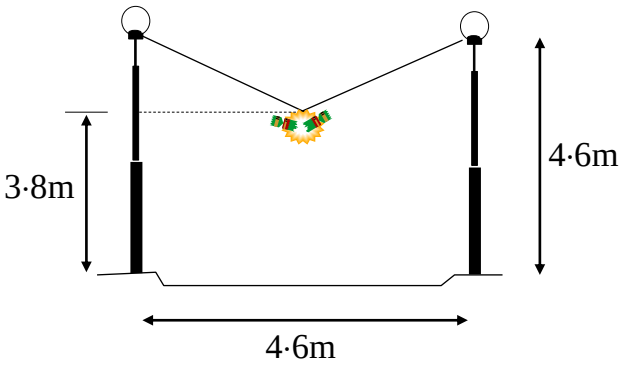


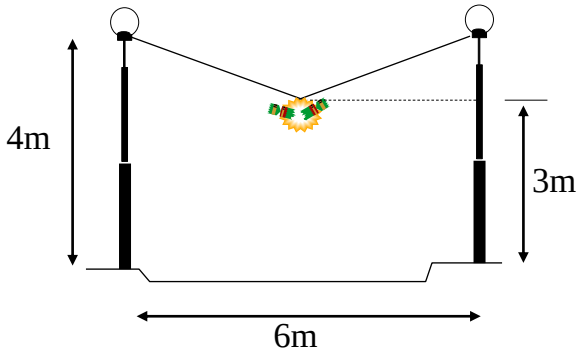
2. Calculate the perimeter of each shape below.





3. Blackpool's light decorations are suspended above the street by wire cables as shown below.



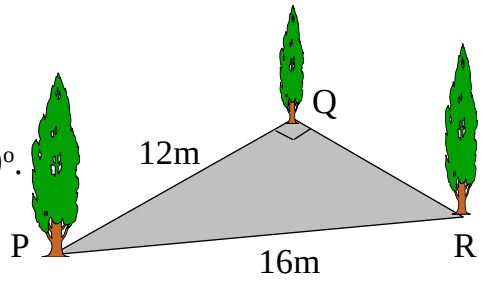


Calculate the total length of cable in each diagram.

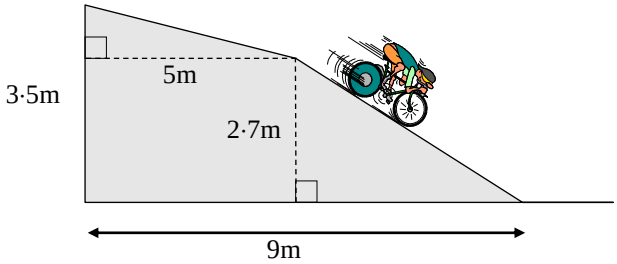
4. Three trees are situated as shown with angle PQR = 90°.

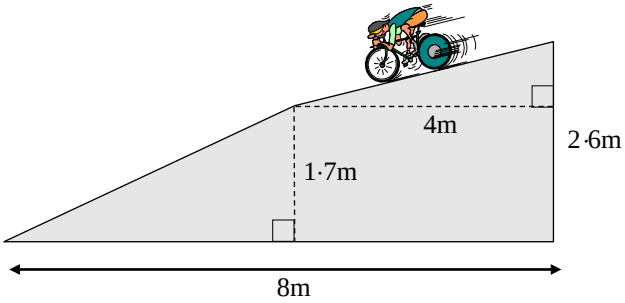
Calculate the distance between the trees Q and R.

(careful !)



5. Calculate the total length of each mountain bike ramp shown below.





Solutions

Applying Geometric Skills to Sides and Angles of a Shape

Using Pythagoras' Theorem

1. **(a)** 10 **(b)** 17 **(c)** 25 **(d)** 10·3 **(e)** 14·7 **(f)** 8·1

(g) 6·6 **(h)** 2·8 **(i)** 124 **(j)** 2·0 **(k)** 17·4 **(l)** 16·6

(m) 68·5 **(n)** 22·7 **(o)** 4·0

2. **(a)** 6·2 **(b)** 12·7 **(c)** 23 **(d)** 8 **(e)** 10·3 **(f)** 6·8

(g) 4·8 **(h)** 2·4 **(i)** 62 **(j)** 2·1 **(k)** 12·1 **(l)** 13·3

(m) 42·5 **(n)** 16·4 **(o)** 1·5

3. 2·4 m **4**. 2·5 m **5**. 3·8 m

6. 13·1 m **7**. 83·2 cm **8.** **(a)** 11·7cm **(b)** 12·7cm

9. **(a)** 20cm **(b)** 15cm

10. **(a)** 15cm **(b)** 16cm

11. 19·2km **12.** 427.2km **13.** 9·6km

14. 7·9km **15.** 15·6mm **16.** 16·6cm

17. 4·6m **18.** 4·4m

19. 16·9m **20.** 1·92m

Pythagoras' Theorem (more practice)

1. 6·7m

2. P = 46·4cm; P = 46·4m

3. 4·8m or 4·9m depending on rounding; 6·3m or 6·4m depending on rounding

4. 10·6m

5. 9·9m; 8·4 or 8·5m depending on rounding.