Lesmahagow High School

Mathematics Department

S3

Equation of a Straight Line

DETERMINING the EQUATION of a STRAIGHT LINE

1. For each line, write down the gradient and the coordinates of the point where it crosses the y – axis.

(a) $y = 3x + 1$ **(b)** $y = \frac{1}{2}x - 5$ **(c)** $y = -2x + 3$

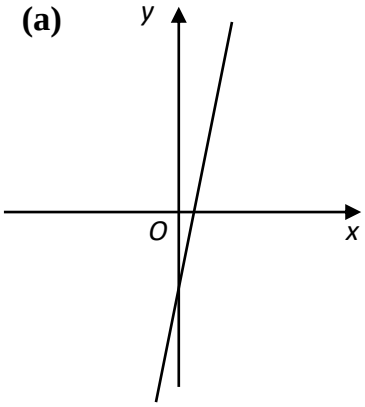
(d) $y = -\frac{1}{4}x - 2$ **(e)** $y = 8x - \frac{1}{2}$ **(f)** $y = -x + 4$

2. Match these equations with the graphs shown below.

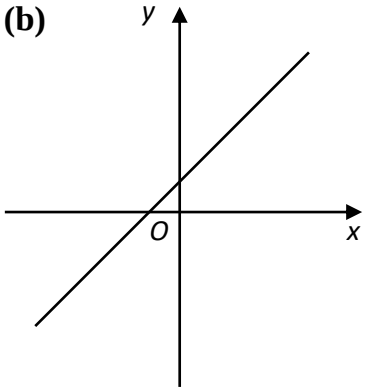
1. $y = x + 1$ **2**. $y = -2x - 3$ **3**. $y = \frac{1}{2}x + 4$

4. $y = -\frac{1}{4}x + 2$ **5**. $y = 6x - 2$ **6**. $y = 3x - 5$

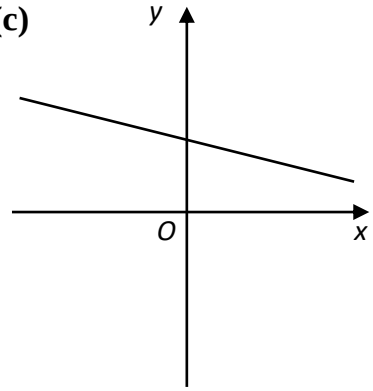
(a)



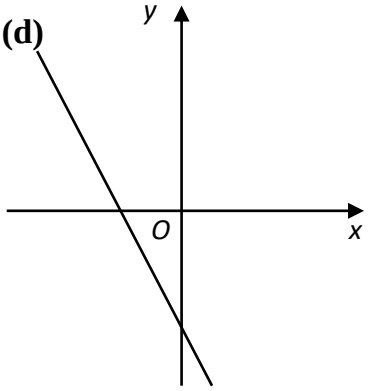
(b)



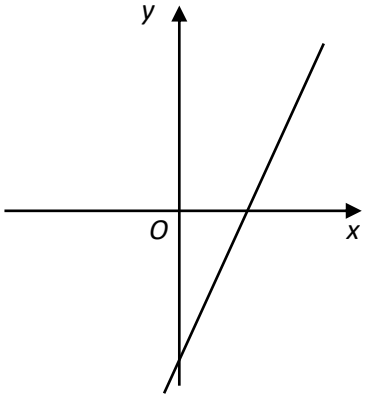
(c)



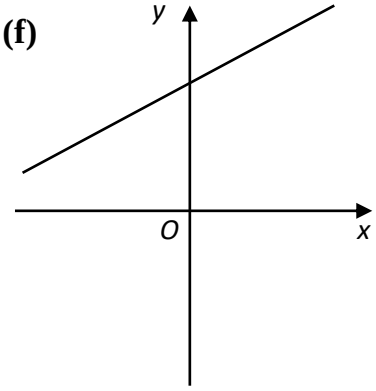
(d)



(e)



(f)



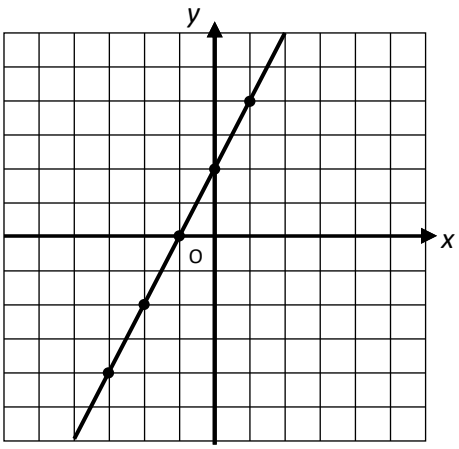
3. Sketch the graphs of lines with equations:

(a) $y = \frac{1}{2}x - 2$ **(b)** $y = -2x - 1$ **(c)** $y = -3x + 2$

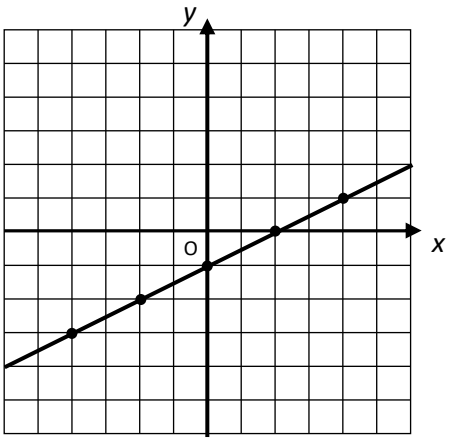
(d) $y = -x + 3$ **(e)** $y = 2x + 3$ **(f)** $y = 4x + 1$

4. Write down the equation of the lines drawn in the diagrams below.

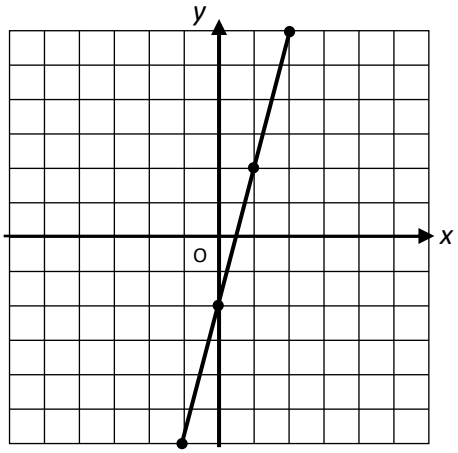
(a)



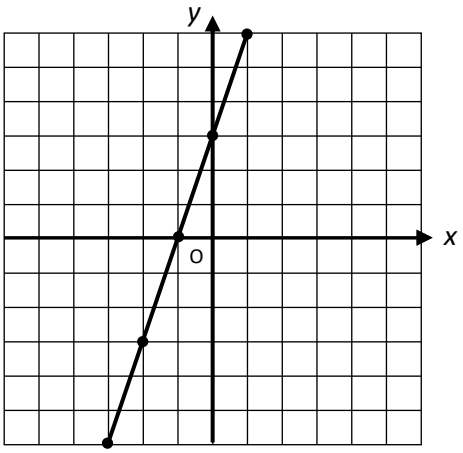
(b)



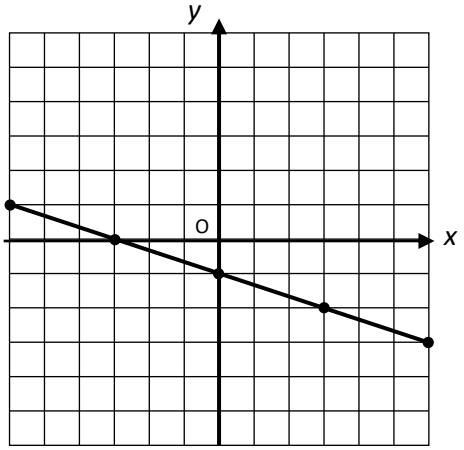
(c)



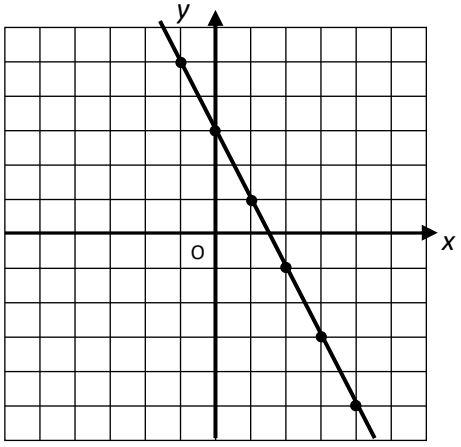
(d)



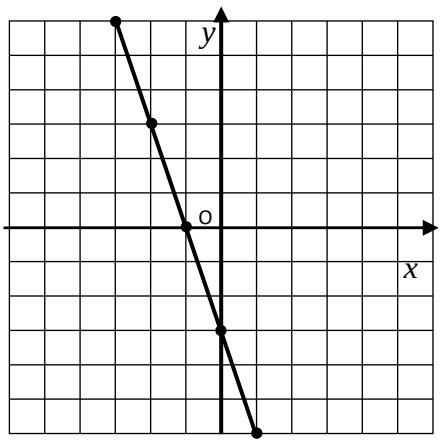
(e)



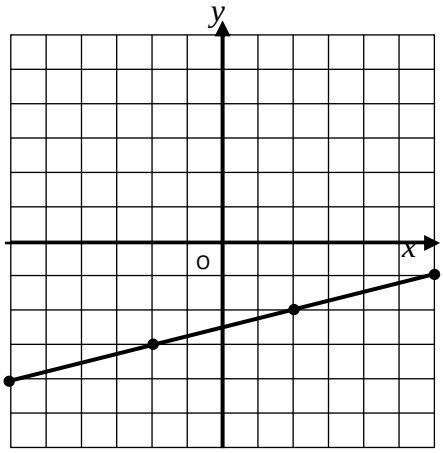
(f)



(g)



(h)



5. Identify the gradient and y – intercept of these lines.

(a) $y = x + 3$ **(b)** $y = -2x - 1$ **(c)** $y = \frac{1}{2}x$

(d) $y = -\frac{1}{2}x + 2$ **(e)** $x + y = 6$ **(f)** $2y = x - 4$

(g) $3y = x + 12$ **(h)** $4x + 5y = 20$ **(i)** $3x - 2y = 12$

6. State the gradient and the y – intercept for each line below.

(a) $y = x - 7$ **(b)** $y = -5x + 3$ **(c)** $5y = 3x - 10$

(d) $y = -4x$ **(e)** $2x + y = 11$ **(f)** $2y = x - 5$

(g) $3y - x = 18$ **(h)** $3x + 7y - 21 = 0$ **(i)** $4x - 5y = 20$

7. Write down the equation of the lines described below:

(a) with gradient 4, passing through the point (0, 5)

(b) with gradient –2, passing through the point (0, 1)

(c) with gradient $\frac{3}{4}$, passing through the point (0, –3)

(d) with gradient 4, passing through the point (3, 1)

(e) with gradient –5, passing through the point (–3, 1)

(f) with gradient $\frac{1}{2}$, passing through the point (–5, –2)

(g) with gradient $\frac{4}{3}$, passing through the point (2, 7)

(h) with gradient $-\frac{3}{4}$, passing through the point (–2, –2)

(i) with gradient $-\frac{3}{2}$, passing through the point (–5, 3)

8. Find the equation of the line joining each pair of points below.

(a) A(4, 3) and B(8, 11) **(b)** C(1, 9) and D(3, 1) **(c)** E(–2, 6) and F(8, 8)

(d) G(5, –9) and H(8, –15) **(e)** I(0, 6) and J(5, 11) **(f)** K(–1, –3) and L(7, –9)

(g) M(–4, 0) and N(–1, 5) **(h)** P(2, 2) and Q(–3, 4) **(i)** R(5, –1) and S(–2, 10)

9. Find the equations of the lines joining the following pairs of points:

(a) (2, 1) and (6, 3) **(b)** (1, 5) and (3, 1) **(c)** (2, 0) and (4, 6)

(d) (–2, –3) and (2, 3) **(e)** (–1, 2) and (5, –1) **(f)** (–4, 2) and (4, –4)

(g) (–6, –2) and (–5, 3) **(h)** (4, –3) and (6, 5) **(i)** (–2, 3) and (0, –2)

10. Establish the equation of the line passing through each pair of points below.

(a) A(2, 1) and B(6, 13) **(b)** C(3, 4) and D(5, –4) **(c)** E(–2, –1) and F(6, 3)

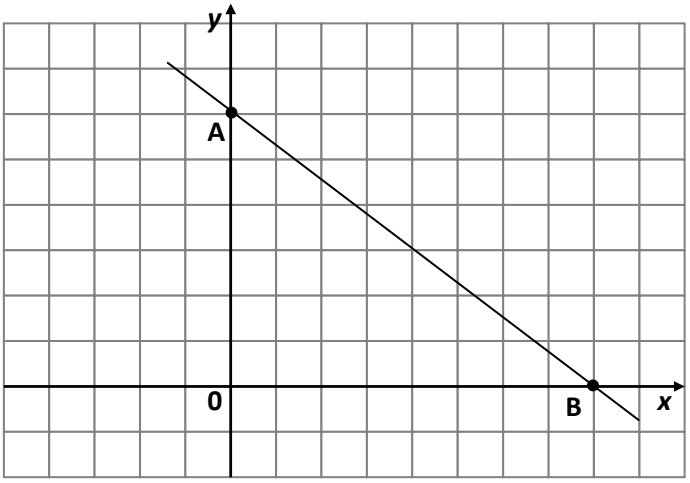
(d) G(4, –13) and H(–2, –1) **(e)** I(2, 8) and J(10, 12) **(f)** K(–3, 2) and L(9, –2)

STRAIGHT LINE **EXAM QUESTIONS**

1. A straight line has the equation $3x - 2y = -4$.

Find the gradient and y-intercept of the line.

2. The line AB passes through the points (0, 6) and (8, 0) as shown in the diagram.



Find the equation of the line AB.

3. A straight line has equation $2y + 3x = 8$. Which line of these gives its gradient and y – intercept? Show working to explain your answer.

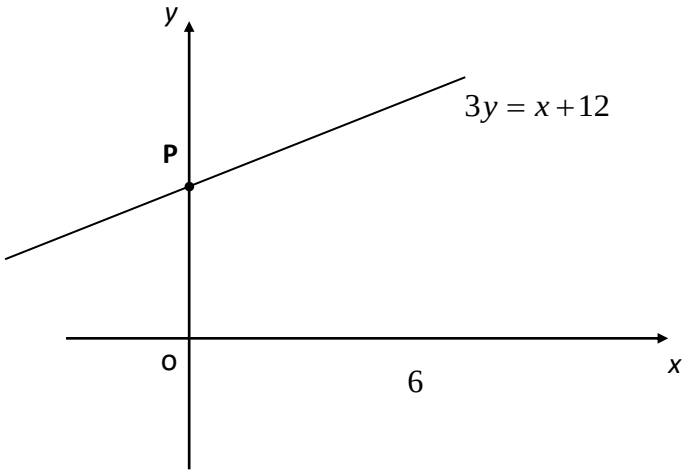
A. 3 and (0, 8) **B.** – 3 and (0. 8)

C. $\frac{3}{2}$ and (0, 4) **D.** $-\frac{3}{2}$ and (0, 4)

4. Find the gradient and y – intercept of the straight line with equation

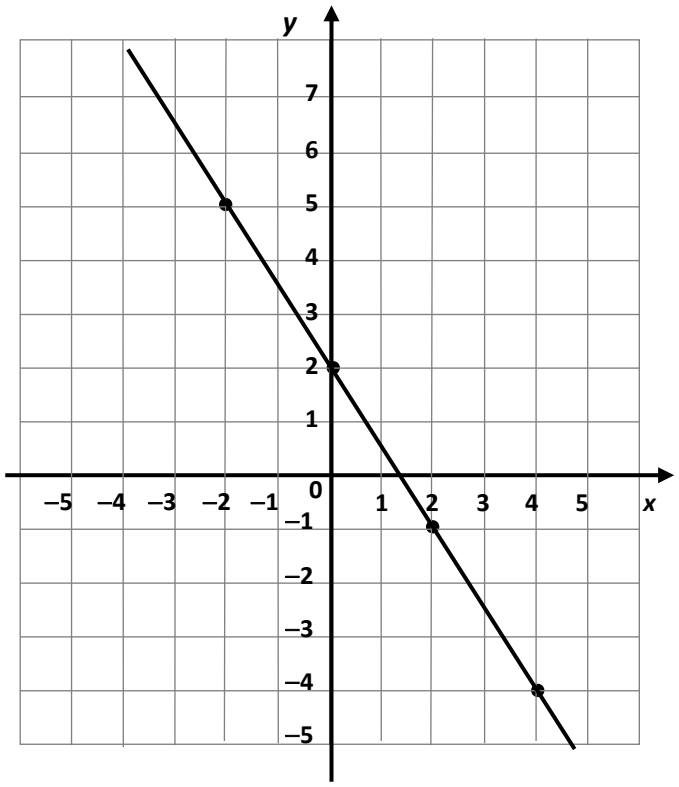
$$3x - 4y = 12.$$

5. The diagram below shows the line with equation $3y = x + 12$.



Find the coordinates of **P**, the point where the line cuts the *y*-axis.

6. Find the equation of the line shown in the diagram below.

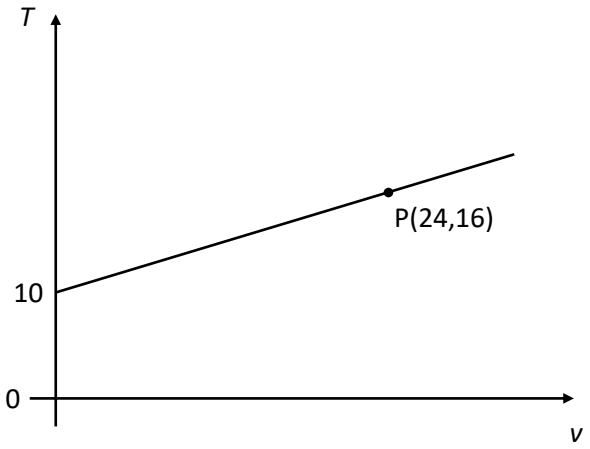


7. A line has equation $2y + 6x = 9$. Find its gradient and y - intercept**.**

8. A line has equation $3y + 4x = 15$. Make a sketch of this line on plain paper showing clearly where it crosses the y - axis.

9. The relationship between variables v and T produces a straight line graph as shown below.

The line passes through the point P(24,16) as shown.



(a) Find the gradient of the line.

(b) Hence, write down the equation of the line in terms of v and T.

10. A straight line has equation $3y - 2x = 6$. Find the gradient and y-intercept of the line.

11. A straight line has equation $3x - 2y = 8$. Find the gradient and y-intercept of the line.

12. Find the equation of the straight line which passes through the point A(3, –2) and is parallel to the line $3y - 2x = 5$

13. **(a)** A straight line has equation $4y - 3x = 6$.

State the gradient and the y-intercept point for this line.

(b) Write down the equation of the line with gradient $-\frac{1}{2}$ which has the same y – intercept point as the line above.

14. **(a)** A straight line has equation $3y - 4x = 12$.

State the gradient and the y-intercept point for this line.

(b) Write down the equation of the line with gradient $-\frac{3}{4}$ which has the same y – intercept point as the line above.

DETERMINING the EQUATION of a STRAIGHT LINE

1. **(a)** 3; (0, 1) **(b)** ½; (0, – 5) **(c)** –2; (0, 3)
(d) –¼; (0, – 2) **(e)** 8; (0, – ½) **(f)** –1; (0, 4)

2. **1** and **(b)** **2** and **(d)** **3** and **(f)** **4** and **(c)** **5** and **(a)** **6** and **(e)**

3. Lines sketched**.**

4. **(a)** $y = 2x + 2$ **(b)** $y = \frac{1}{2}x - 1$ **(c)** $y = 4x - 2$ **(d)** $y = 3x + 3$

(e) $y = -\frac{1}{3}x - 1$ **(f)** $y = -\frac{1}{2}x + 3$ **(g)** $y = -3x - 3$ **(h)** $y = \frac{1}{4}x - 2 \cdot 5$

5. **(a)** 1; (0, 3) **(b)** $-2;\ (0,-1)$ **(c)** $\frac{1}{2};\ (0,0)$

(d) $-\frac{1}{2};\ (0,\ 2)$ **(e)** $-1;\ (0,6)$ **(f)** $\frac{1}{2};(0,\ -2)$

(g) $\frac{1}{3};(0,\ 4)$ **(h)** $-\frac{4}{5};(0,\ 4)$ **(i)** $\frac{3}{2};(0,\ -6)$

6. **(a)** $1;\ (0,-7)$ **(b)** $-5;\ (0,3)$ **(c)** $\frac{3}{5};(0,\ -2)$

(d) $-4;\ (0,0)$ **(e)** $-2;(0,\ 11)$ **(f)** $\frac{1}{2};(0,\ -\frac{5}{2})$

(g) $\frac{1}{3};(0,\ 6)$ **(h)** $-\frac{3}{7};(0,\ 3)$ **(i)** $\frac{4}{5};(0,\ -4)$

7. **(a)** $y = 4x + 5$ **(b)** $y = -2x + 1$ **(c)** $y = \frac{3}{4}x - 3$

(d) $y = 4x - 11$ **(e)** $y = -5x - 14$ **(f)** $2y - x = 1$

(g) $3y - 4x = 13$ **(h)** $3x + 4y = -14$ **(i)** $2x + 3y = -9$

8. **(a)** $y = 2x - 5$ **(b)** $y + 4x = 13$ **(c)** $5y = x + 32$

(d) $y = 2x - 1$ **(e)** $y = x + 6$ **(f)** $4x + 3y = -15$

(g) $3y - 5x = 20$ **(h)** $2x + 5y = 14$ **(i)** $7y + 11x = 48$

9. **(a)** $2y - x = 0$ **(b)** $y + 2x = 7$ **(c)** $y = 3x - 6$

(d) $2x-3y=0$ **(e)** $2y+x=3$ **(f)** $4y+3x=-4$

(g) $y=5x+28$ **(h)** $y=4x-19$ **(i)** $2y+5x=-4$

10. **(a)** $y=3x-5$ **(b)** $y+4x=16$ **(c)** $2y-x=0$

(d) $y+2x=-5$ **(e)** $2y-x=14$ **(f)** $3y+x=3$

STRAIGHT LINE EXAM QUESTIONS

1. $\frac{3}{2}$; (0, 2) **2.** $y=-\frac{3}{4}x+6$ **3.** D **4.** $\frac{3}{4}$; (0, 3)

5. P(0, 4) **6.** $y=-\frac{3}{2}x+2$ **7.** -3; $(0, 4\cdot5)$

8. Line crossing at (0, 5) with gradient $-\frac{4}{3}$ **9.** **(a)** $\frac{1}{4}$ **(b)** $T=\frac{1}{4}v+10$

10. $\frac{2}{3}$; (0, 2) **11.** $\frac{3}{2}$; (0, −4) **12.** $3y-2x=-12$

13. **(a)** $\frac{3}{4}$; $(0, 1\cdot5)$ **(b)** $2y+x=3$ **14.** **(a)** $\frac{4}{3}$; (0, 4) **(b)** $4y+3x=16$

1.2 WORKING with LINEAR EQUATIONS and INEQUATIONS

1.	**(a)**	2	**(b)**	4	**(c)**	3	**(d)**	5	**(e)**	2	**(f)** 5
	(g)	4	**(h)**	1	**(i)**	16	**(j)**	16	**(k)**	15	**(l)** 7
	(m)	10	**(n)**	20	**(o)**	17	**(p)**	19			
2.	**(a)**	3	**(b)**	4	**(c)**	2	**(d)**	9	**(e)**	4	**(f)** 4
	(g)	2	**(h)**	5	**(i)**	4	**(j)**	9	**(k)**	10	**(l)** 8
	(m)	7	**(n)**	5	**(o)**	7	**(p)**	7			
3.	**(a)**	−18	**(b)**	11	**(c)**	−8	**(d)**	16	**(e)**	−13	**(f)** 9
	(g)	−12	**(h)**	25	**(i)**	−9	**(j)**	8·5	**(k)**	−5·5	**(l)** 5·5
	(m)	−3·5	**(n)**	18	**(o)**	−4·2	**(p)**	10·5			
4.	**(a)**	4	**(b)**	1	**(c)**	−5	**(d)**	1	**(e)**	−6	**(f)** 3
	(g)	7	**(h)**	7	**(i)**	7	**(j)**	0	**(k)**	20	**(l)** −10
	(m)	4	**(n)**	7	**(o)**	2	**(p)**	5			

5. **(a)** 3 **(b)** 4 **(c)** 6 **(d)** 4 **(e)** 2 **(f)** 5

(g) 6 **(h)** 8 **(i)** 8 **(j)** 8 **(k)** 12 **(l)** 9

(m) 6 **(n)** 7 **(o)** 15 **(p)** 18

6. **(a)** 1 **(b)** 2 **(c)** 6 **(d)** 5 **(e)** 3 **(f)** 2

(g) 5 **(h)** 6 **(i)** 11 **(j)** 9 **(k)** 11 **(l)** 20

(m) 7 **(n)** 12 **(o)** 21

7. **(a)** 3 **(b)** 2 **(c)** 2 **(d)** 3 **(e)** 2 **(f)** 3

(g) 6 **(h)** 5 **(i)** 3 **(j)** 6 **(k)** 4 **(l)** 0·5

(m) 8 **(n)** 11 **(o)** 15 **(p)** 3 **(q)** 12 **(r)** 4

(s) 7 **(t)** 16 **(u)** 9 **(v)** 7 **(w)** 4 **(x)** 2

8. **(a)** 6 **(b)** 6 **(c)** 4 **(d)** 1 **(e)** 4 **(f)** 6

(g) 4 **(h)** 24 **(i)** 8 **(j)** 7 **(k)** 5 **(l)** 2·5

(m) 2·5 **(n)** 16 **(o)** 15 **(p)** 2 **(q)** 2 **(r)** 4

(s) 7 **(t)** 6 **(u)** 2 **(v)** 7 **(w)** 12 **(x)** 40

9. **(a)** $x > 1$ **(b)** $x > 3$ **(c)** $x > 4$ **(d)** $x > 4$ **(e)** $a > 3$ **(f)** $y > 3$

(g) $p > 9$ **(h)** $c > 1$ **(i)** $b > 6$ **(j)** $q > 0$ **(k)** $d > 3$ **(l)** $x > 4$

(m) $c > 5$ **(n)** $p > 9$ **(o)** $a > 12$ **(p)** $y > 12$

10. **(a)** $x < 2$ **(b)** $x < 7$ **(c)** $x < 10$ **(d)** $x < 4$ **(e)** $a < 3$ **(f)** $y < 6$

(g) $p < 8$ **(h)** $c < 4$ **(i)** $b < 5$ **(j)** $q < 17$ **(k)** $d < 0$ **(l)** $x < 5$

(m) $c < 6$ **(n)** $p < 14$ **(o)** $a < 11$ **(p)** $y < 1$

11. **(a)** $x > 3$ **(b)** $x > 4$ **(c)** $x > 4$ **(d)** $x > 9$ **(e)** $a > 4$ **(f)** $y > 4$

(g) $p > 3$ **(h)** $c > 5$ **(i)** $b < 4$ **(j)** $q < 9$ **(k)** $d < 10$ **(l)** $x < 8$

(m) $c < 7$ **(n)** $p < 5$ **(o)** $a < 7$ **(p)** $y < 7$

12. **(a)** $x < 7$ **(b)** $x > 6$ **(c)** $x > 11$ **(d)** $x < 9$ **(e)** $a < 6$ **(f)** $y > 11$

(g) $p < 18$ **(h)** $c > 9$ **(i)** $b > 16$ **(j)** $q < 16$ **(k)** $d > 15$ **(l)** $x > 7$

(m) $c > 10$ **(n)** $p < 20$ **(o)** $a < 17$ **(p)** $y < 19$

13. **(a)** $x < 2$ **(b)** $x > 2$ **(c)** $x > 3$ **(d)** $x > 2$ **(e)** $a < 3$ **(f)** $y < 2$

(g) $p > 8$ **(h)** $c < 5$ **(i)** $b > 6$ **(j)** $q < 0$ **(k)** $d < 10$ **(l)** $x > 4$

(m) $c < 3$ **(n)** $p < 7$ **(o)** $a > 2$ **(p)** $y < 3$

14. **(a)** $x > 3$ **(b)** $x > 4$ **(c)** $x < 6$ **(d)** $x > 3$ **(e)** $a < 2$ **(f)** $y < 5$

(g) $p > 6$ **(h)** $c > 8$ **(i)** $b > 8$ **(j)** $q < 8$ **(k)** $d < 12$ **(l)** $x > 9$

(m) $c < 6$ **(n)** $p < 7$ **(o)** $a < 15$ **(p)** $y < 18$

15. **(a)** $\{-2, -1, 0, 1\}$ **(b)** $\{-2, -1\}$ **(c)** $\{5\}$ **(d)** $\{-2, -1\}$

(e) $\{-2, -1, 0\}$ **(f)** $\{1,2,3,4,5\}$ **(g)** $\{-2, -1, 0, 1, 2\}$ **(h)** $\{1,2,3,4,5\}$

16. **(a)** $a \leq 3$ **(b)** $x > 1$ **(c)** $p \geq 2$ **(d)** $k < -3$

(e) $m \leq 7$ **(f)** $y > 2 \cdot 9$ **(g)** $h < 1$ **(h)** $x > \frac{1}{5}$

17. **(a)** $a \geq 3$ **(b)** $x < 2$ **(c)** $p \leq 2$ **(d)** $k > -2$

(e) $d \geq \frac{38}{9}$ **(f)** $y < -\frac{10}{3}$ **(g)** $h > 0$ **(h)** $y < \frac{11}{9}$

18. {0, 1, 2, 3, 4, 5} **19.** {0, 1, 2} **20.** Jane must be younger than 11

1.3 WORKING WITH SIMULTANEOUS EQUATIONS

GRAPHICAL SOLUTION

1. **(a)** Tables completed Table 1: 9, 6, 2 Table 2: 1, 4, 6

(b) and **(c)** Graphs drawn

(d) (5, 4)

2. **(a)** Tables completed Table 1: 0, 5, 1 Table 2: 0, 3, 5

(b) and **(c)** Graphs drawn

(d) (5, 3)

3. **(a)** (4, 3) **(b)** (11, 3) **(c)** (9, 6) **(d)** (12, 5)

(e) (8, 4) **(f)** (20, 10) **(g)** (15, 3) **(h)** (8, 3)

(i) (7, 3) **(j)** (13, 4)

4. **(a)** (2, 4) **(b)** (2, 3) **(c)** (3, 1) **(d)** (3, 2)

(e) (4, 3) **(f)** (1, 5)

5. **(a)** (6, 5) **(b)** (6, 2) **(c)** (2, 4) **(d)** (2, 2)

(e) (10, 3) **(f)** (4, –3) **(g)** (2, 5) **(h)** (2, 0)

(i) (1, –1) **(j)** (4, 6) **(k)** (2, 0) **(l)** (–6, 6)

(m) (5, –2) **(n)** (–1, –3) **(o)** (1, –1)

ALGEBRAIC SOLUTION

1. **(a)** $x = 5$ and $y = 5$ **(b)** $x = 1$ and $y = 1$ **(c)** $x = 2$ and $y = 4$

(d) $x = 3$ and $y = 6$ **(e)** $x = 3$ and $y = 10$ **(f)** $x = 5$ and $y = 21$

2. **(a)** $x = 3$ and $y = 1$ **(b)** $x = 7$ and $y = 2$ **(c)** $x = 5$ and $y = 2$

(d) $x = 2$ and $y = –1$ **(e)** $x = 6$ and $y = –3$ **(f)** $x = 4$ and $y = –5$

(g) $x = –3$ and $y = –2$ **(h)** $x = –11$ and $y = –3$ **(i)** $x = –8$ and $y = –10$

3. **(a)** $x = 2$ and $y = 5$ **(b)** $a = 3$ and $b = 2$ **(c)** $e = 6$ and $f = 2$

(d) $x = –1$ and $y = 3$ **(e)** $x = 2$ and $y = –2$ **(f)** $p = –2$ and $q = 3$

(g) $g = –4$ and $h = 3$ **(h)** $x = 3$ and $y = 4$ **(i)** $u = –2$ and $v = –3$

(j) $x = 4$ and $y = 1$ **(k)** $p = –5$ and $q = 2$ **(l)** $a = 4$ and $b = –2$

4. **(a)** $x = 2$ and $y = 5$ **(b)** $a = 3$ and $b = –1$ **(c)** $e = –2$ and $f = 5$

(d) $x = –1$ and $y = 4$ **(e)** $x = 3$ and $y = 4$ **(f)** $p = –3$ and $q = 4$

(g) $g = 8$ and $h = –5$ **(h)** $x = 7$ and $y = –2$ **(i)** $u = 2$ and $v = –2$

(j) $x = 4$ and $y = 1$ **(k)** $p = –1$ and $q = 2$ **(l)** $a = 8$ and $b = –2$

5. **(a)** $x = 7$ and $y = 1$ **(b)** $x = 10$ and $y = 1$ **(c)** $x = 4$ and $y = 2$

(d) $x = 2$ and $y = 3$ **(e)** $x = 3$ and $y = –1$ **(f)** $x = 4$ and $y = 4$

(g) $x = 2$ and $y = 5$ **(h)** $x = 5$ and $y = 1$ **(i)** $x = 2$ and $y = 1$

(j) $x = 4$ and $y = 2$ **(k)** $x = –1$ and $y = –2$ **(l)** $x = 1$ and $y = –1$

6. **(a)** $x = 1$ and $y = 2$ **(b)** $x = 3$ and $y = –1$ **(c)** $x = 2$ and $y = 2$

(d) $x = 5$ and $y = 1$ **(e)** $x = 5$ and $y = 7$ **(f)** $x = 5$ and $y = –2$

(g) $x = 3$ and $y = –3$ **(h)** $x = 4$ and $y = –3$ **(i)** $x = 2$ and $y = 3·5$

(j) $x = 0{\cdot}5$ and $y = 2$ **(k)** $x = 1$and $y = 2$ **(l)** $x = 5$ and $y = 0$

WORKING with SIMULTANEOUS EQUATIONS in CONTEXT

1. 36 and 20 **2.** 18 and 4 **3.** 8 and 5 **4.** 9 and 2

5. Chocolate costs 40p and crisps cost 30p

6. Sandwich costs £1.20 and hotdog costs 90p.

7. Ruler costs 49p and pencil costs 26p.

8. Download is £9 and CD is £13

9. Standard print is 21p and Jumbo costs 55p.

10. Centre costs £11.25 and Side costs £9.50

11. Large glass holds 145ml and small holds 95ml.

12. Frame tent holds 8 and ridge tent holds 3.

13. Reader's letter pays £15 and Star letter pays £25.

14. Small takes 1·8kg and the large takes 2kg.

15. Thursday should have been £21.95.

16. **(a)** $4x + 4y = 60$; $6x + 16y = 120$; $x = 12$ and $y = 3$ **(b)** 144cm²

17. **(a)** Box weighs 6kg and parcel weighs 2kg. **(b)** 58kg

18. Milk is 24p and butter is 96p.

19. 38 hours basic and 7 hours overtime

20. 320 cheaper tickets were sold

21. 33 × 20p coins and 21 × 50p coins

WORKING WITH SIMULTANEOUS EQUATIONS EXAM QUESTIONS

1. £21

2. **(a)** $s + b = 640$ **(b)** $8{\cdot}5s + 12{\cdot}2b = 6143$

(c) 450 stalls tickets and 190 balcony tickets

3. £5.02 **4.** £81.70 **5.** (2, –3)

6. **(a)** $f + t = 60$ **(b)** $25f + 20t = 1325$

(c) Clare sold 35 treacle scones

7. 75 points are needed – tokens give only 70 points so not enough.

8. £11.01. **9.** $x = 3$ and $y = 2$ **10.** (2, –3)

11. £9.72 **12.** Sofa costs £425 and chair costs £295.

13. £23.87 **14.** £25.60 **15.** (–2, 5)

1.4 CHANGING the SUBJECT of a FORMULA

1. **(a)** $x = y - 3$ **(b)** $x = y + 5$ **(c)** $x = y - a$ **(d)** $x = y + b$

(e) $x = \frac{y}{3}$ **(f)** $x = \frac{y}{10}$ **(g)** $x = \frac{y}{k}$ **(h)** $x = \frac{y}{a}$

(i) $x = y - 3p$ **(j)** $x = y + 5t$ **(k)** $x = \frac{y-1}{2}$ **(l)** $x = \frac{y+7}{3}$

(m) $x = \frac{y-4a}{7}$ **(n)** $x = \frac{y-38}{4}$ **(o)** $x = \frac{y-8}{10}$

2. **(a)** $a = 4 - b$ **(b)** $a = 12 - d$ **(c)** $a = 5x - y$ **(d)** $a = \frac{2-m}{2}$

(e) $a = \frac{7-q}{5}$ **(f)** $a = \frac{20-c}{3}$ **(g)** $a = \frac{s-r}{2}$ **(h)** $a = \frac{d-t}{4}$

(i) $a = \frac{4b-z}{5}$ **(j)** $a = \frac{2h-k}{7}$ **(k)** $a = \frac{6p-q}{11}$ **(l)** $a = \frac{2x-g}{9}$

3. **(a)** $x = \frac{y-b}{a}$ **(b)** $x = \frac{y-c}{m}$ **(c)** $x = \frac{t+r}{s}$ **(d)** $x = \frac{p-2r}{q}$

(e) $x = \frac{m+3n}{f}$ **(f)** $x = \frac{a-b}{c}$ **(g)** $x = \frac{h-k}{m}$ **(h)** $x = \frac{d-3b}{c}$

(i) $x = \frac{kc-g}{h}$

4. **(a)** $l = \frac{P}{4}$ **(b)** $I = \frac{V}{R}$ **(c)** $T = \frac{S}{D}$ **(d)** $b = \frac{A}{l}$

(e) $d = \frac{C}{\pi}$ **(f)** $U = \frac{G}{T}$ **(g)** $t = \frac{v-u}{a}$ **(h)** $l = \frac{P-2b}{2}$

(i) $y = \frac{H-5m}{x}$

5. **(a)** $c = 2b$ **(b)** $c = 5x$ **(c)** $c = 4y$ **(d)** $c = 6m$

(e) $c = 9k$ **(f)** $c = 10d$ **(g)** $c = 2a - 4$ **(h)** $c = 3h + 15$

(i) $c = 4p - 4q$ **(j)** $c = 10y + 10x$ **(k)** $c = 8t - 16s$ **(l)** $c = 5r + 15q$

6. **(a)** $x = \frac{3}{y}$ **(b)** $x = \frac{c}{d}$ **(c)** $x = \frac{y}{m}$ **(d)** $x = \frac{a+2}{s}$

(e) $x = \frac{z-1}{w}$ **(f)** $x = \frac{b+c}{a}$ **(g)** $x = 9a - 8$ **(h)** $x = 2k + 5$

(i) $x = 3 - 4p$ **(j)** $x = \frac{2}{y-1}$ **(k)** $x = \frac{6}{z-7}$ **(l)** $x = \frac{m}{h-k}$

7. **(a)** $k = y^2$ **(b)** $k = x^2$ **(c)** $k = m^2$ **(d)** $k = a^2 b$

(e) $k = c^2 d$ **(f)** $k = gh^2$ **(g)** $k = \frac{t}{s^2}$ **(h)** $k = \frac{p}{q^2}$

(i) $k = \frac{z}{w^2}$ **(j)** $k = \sqrt{r}$ **(k)** $k = \sqrt{ab}$ **(l)** $k = \sqrt{\frac{p}{q}}$

(m) $k = \sqrt{y-x}$ **(n)** $k = \sqrt{c+d}$ **(o)** $k = \sqrt{\frac{x+1}{3}}$

8. **(a)** $s = \frac{v^2 - u^2}{2a}$ **(b)** $u = \sqrt{v^2 - 2as}$ **(c)** $h = \frac{V}{\pi r^2}$ **(d)** $r = \sqrt{\frac{V}{\pi h}}$

(e) $A = \pi r^2$ **(f)** $a = \frac{(L-3)^2}{6}$ **(g)** $p = 4k^2 - 4$ **(h)** $y = \frac{tx^2}{4z}$

(i) $b = \frac{x}{4a^2 r^2}$ **(j)** $A = \sqrt{\frac{st}{x-3y}}$ **(k)** $x = \frac{R + 3A^2 y}{A^2}$ **(l)** $n = \sqrt{\frac{1}{a^2+1}}$

(m) $n = \frac{t}{t-d}$ **(n)** $R = \frac{r_1 r_2}{r_2 + r_1}$ **(o)** $a = \sqrt{\frac{4d}{x+b}}$

CHANGING the SUBJECT of a FORMULA — EXAM QUESTIONS

1. $\sqrt{\frac{x}{4a^2b^2}}$ or $\frac{\sqrt{x}}{2ab}$ **2.** $g = \frac{V^2}{2R}$ **3.** $x = (A^2 - 5)^2$

4. $v = \sqrt{\frac{2E}{m}}$ **5.** $a = \sqrt{\frac{v}{3b}}$ **6.** $k = \frac{4\pi^2 m}{T^2}$

7. $C = \frac{5}{9}(F - 32)$ **8.** $r = \sqrt{\frac{3V}{\pi h}}$ **9.** $k = \sqrt{mab - 2}$

10. $b = \sqrt{\frac{a-7}{c}}$ **11.** $m = \frac{3k}{n}$ **12.** $r = \sqrt[3]{\frac{3V}{4\pi}}$

2.1 RECOGNISE and DETERMINE the EQUATIONS of QUADRATICS from their GRAPHS

1. **(a)** $y = x^2$ **(b)** $y = 3x^2$ **(c)** $y = 5x^2$ **(d)** $y = 1 \cdot 5x^2$

(e) $y = 5x^2$ **(f)** $y = 3x^2$ **(g)** $y = -x^2$ **(h)** $y = -2x^2$

(i) $y = -5x^2$ **(j)** $y = \frac{1}{2}x^2$ **(k)** $y = \frac{1}{4}x^2$ **(l)** $y = \frac{1}{3}x^2$

(m) $y = 40x^2$ **(n)** $y = -25x^2$ **(o)** $y = -\frac{3}{4}x^2$

2. **(a)** $y = x^2 + 2$ **(b)** $y = x^2 - 1$ **(c)** $y = x^2 + 1 \cdot 5$ **(d)** $y = -x^2 + 5$

(e) $y = -x^2 + 3$ **(f)** $y = -x^2 - 2$ **(g)** $y = 2x^2 + 1$ **(h)** $y = 5x^2 + 4$

(i) $y = 3x^2 + 2$ **(j)** $y = 2x^2 - 3$ **(k)** $y = \frac{1}{2}x^2 - 9$ **(l)** $y = -2x^2 + 8$

(m) $y = -x^2 + 3$ **(n)** $y = -3x^2 - 2$

3. **(a)** $y = (x-2)^2 + 1$ **(b)** $y = (x-1)^2 + 6$ **(c)** $y = (x-4)^2$

(d) $y = (x-3)^2 - 4$ **(e)** $y = x^2 - 5$ **(f)** $y = (x+1)^2 + 3$

(g) $y = (x+2)^2 - 4$ **(h)** $y = (x+6)^2$ **(i)** $y = (x-4)^2 + 20$

(j) $y = (x-10)^2 - 2$ **(k)** $y = (x-25)^2 + 10$ **(l)** $y = (x+30)^2 + 5$

(m) $y = (x-1)^2 - 1$ **(n)** $y = x^2 + 6$

2.2 SKETCHING a QUADRATIC FUNCTION

1. Graphs should show the following:

(a) Turning point (4, 1); minimum; y – intercept (0, 17)

(b) Turning point (2, 5); minimum; y – intercept (0, 9)

(c) Turning point (1, 7); minimum; y – intercept (0, 8)

(d) Turning point (2, –3); minimum; y – intercept (0, 1)

(e) Turning point (3, –4); minimum; y – intercept (0, 5)

(f) Turning point (5, –2); minimum; y – intercept (0, 23)

(g) Turning point (–4, 6); minimum; y – intercept (0, 22)

(h) Turning point (–1, 5); minimum; y – intercept (0, 6)

(i) Turning point (–8, 1); minimum; y – intercept (0, 65)

(j) Turning point (–3, –1); minimum; y – intercept (0, 8)

(k) Turning point (–½, –¾); minimum; y – intercept (0, –½)

(l) Turning point (–0·5, –2·5); minimum; y – intercept (0, –2·25)

(m) Turning point (1, 4); maximum; y – intercept (0, 3)

(n) Turning point (–6, 3); maximum; y – intercept (0, –33)

(o) Turning point (–7, –2); maximum; y – intercept (0, –51)

(p) Turning point (2, 12); minimum; y – intercept (0, 16)

(q) Turning point (5, –1); minimum; y – intercept (0, 24)

(p) Turning point (4, 3·75); minimum; y – intercept (0, 19·75)

2. Graphs should all be minimum T.P. and show the following points:

(a) (1, 0), (5, 0) and (3, –4)

(b) (2, 0), (4, 0) and (3, –1)

(c) (3, 0), (7, 0) and (5, –4)

(d) (6, 0), (8, 0) and (7, –4)

(e) (2, 0), (5, 0) and (3·5, –2·25)

(f) (5, 0), (8, 0) and (6·5, –2·25)

(g) (−2, 0), (−3, 0) and (−2·5, –0·25)

(h) (−2, 0), (−5, 0) and (−3·5, –2·25)

(i) (−4, 0), (−6, 0) and (−5, –1)

(j) (−3, 0), (−4, 0) and (−3·5, –0·25)

(k) (−9, 0), (−5, 0) and (−7, –4)

(l) (−8, 0), (−3, 0) and (−5·5, –6·25)

3. Graphs should show the following:

(a) (1, 0), (–5, 0), (–2, –9)**,** minimum

(b) (7, 0), (–3, 0), (2, 25)**,** maximum

(c) (5, 0), (–3, 0), (1, 16)**,** minimum

(d) (4, 0), (–8, 0), (–2, 36)**,** maximum

(e) (7, 0), (–1, 0), (3, –16)**,** minimum

(f) (7, 0), (–1, 0), (3, 16)**,** maximum

(g) (3, 0), (–9, 0), (–3, 36)**,** maximum

(h) (10, 0), (–2, 0), (4, –36)**,** minimum

(i) (–7, 0), (9, 0), (1, 64)**,** maximum

(j) (6, 0), (–4, 0), (1, 25)**,** maximum

(k) (1, 0), (–1, 0), (0, 1)**,** maximum

(l) (6, 0), (–2, 0), (2, –16)**,** minimum

(m) (3, 0), (–3, 0), (0, –9)**,** minimum

(n) (7, 0), (–1, 0), (3, 16)**,** maximum

(o) (6, 0), (–10, 0), (–2, 64)**,** maximum

2.3 IDENTIFYING FEATURES of a QUADRATIC FUNCTION

1. **(a)** (1, –2); minimum; $x = 1$

(b) (–2, 3); minimum; $x = –2$

(c) (3, –4); minimum; $x = 3$ **(d)** (–2, –2); minimum; $x = –2$

(e) (1, 6); maximum; $x = 1$ **(f)** (–1, 4); maximum; $x = –1$

(g) (3, –2); maximum; $x = 3$ **(h)** (–3, 3); maximum; $x = –3$

2. (a) (4, 1); minimum; $x = 4$ **(b)** (2, 5); minimum; $x = 2$

(c) (1, 7); minimum; $x = 1$ **(d)** (2, –3); minimum; $x = 2$

(e) (3, –4); minimum; $x = 3$ **(f)** (5, –2); minimum; $x = 5$

(g) (–4, 6); minimum; $x = –4$ **(h)** (–1, 5); minimum; $x = –1$

(i) (–8, 1); minimum; $x = –8$ **(j)** (–3, –1); minimum; $x = –3$

(k) (– ½ ,– ¾); minimum; $x = – ½$ **(l)** (–0·5, – 2·5); minimum; $x = –0·5$

(m) (1, 4); maximum; $x = 1$ **(n)** (–6, 3); maximum; $x = –6$

(o) (–7, –2); maximum; $x = –7$ **(p)** (2, 12); minimum; $x = 2$

(q) (5, –1); minimum; $x = 5$ **(r)** (4, 3·75); minimum; $x = 4$

2.4 WORKING with QUADRATIC EQUATIONS

DRAWING GRAPHS

1. (a) $x = 0$ or 3 **(b)** $x = –3$ or $–2$ **(c)** $x = –2$ or 6 **(d)** $x = –1$ or 8

(e) $x = –10$ or 2 **(f)** $x = –5$ or 1

2. (a) 8, 3, 0, –1, 0, 3, 8; roots 0 and 2 **(b)** 5, 0, –3, –4, –3, 0 5; roots 1 and 5

(c) 8, 3, 0, –1, 0 3, 8; roots –3 and –1 **(d)** 0, 5, 8, 9, 8, 5, 0; roots –4 and 2

3. Graphs drawn with roots:

(a) 0 and 4 **(b)** 0 and –6 **(c)** 0 and 5 **(d)** 5 and 3

(e) –3 (twice) **(f)** 2 (twice) **(g)** –2 and –4 **(h)** –6 and –2

(i) 2 and 5 **(j)** 1 and 4 **(k)** –3 and 2 **(l)** 2 and –1

(m) –6 and 2 **(n)** –1 and` **(o)** –2 and 1

FACTORISING

1. (a) 0 and 5 **(b)** 0 and –7 **(c)** 0 and 1 **(d)** 0 and 3

(e) 0 and –1 **(f)** 0 and 2 **(g)** 2 and 4 **(h)** 3 and 4

(i) 3 and 5 **(j)** –2 and –1 **(k)** –5 and –4 **(l)** –7 and –8

(m) –3 and 1 **(n)** –2 and 12 **(o)** –1 and 9 **(p)** –4 and 4

(q) –7 and 7 **(r)** –5 and 5 **(s)** 4 and ½ **(t)** $-\frac{3}{2}$ and –2

(u) $-\frac{1}{3}$ and $\frac{5}{2}$

2. **(a)** 0 and –4 **(b)** 0 and 2 **(c)** 0 and –8 **(d)** 0 and 1

(e) 0 and –1 **(f)** 0 and –7 **(g)** 0 and –2 **(h)** 0 and 4

(i) 0 and 3 **(j)** 0 and $\frac{3}{2}$ **(k)** 0 or $-\frac{3}{2}$ **(l)** 0 or $-\frac{3}{2}$

(m) 0 and 5 **(n)** 0 and 9 **(o)** 0 and 2

(p) 0 and $\frac{3}{2}$ **(q)** 0 and $\frac{3}{4}$ **(r)** 0 and $\frac{2}{5}$

3. **(a)** –5 and 5 **(b)** –1 and 1 **(c)** –2 and 2 **(d)** –6 and 6

(e) –3 and 3 **(f)** –8 and 8 **(g)** –4 and 4 **(h)** –12 and 12

(i) –10 and 10 **(j)** –7 and 7 **(k)** –9 and 9 **(l)** –11 and 11

(m) –3 and 3 **(n)** –4 and 4 **(o)** –4 and 4

4. **(a)** –3 and –1 **(b)** –5 and –1 **(c)** –7 and –1 **(d)** –3 and –2

(e) –4 and –2 **(f)** –3 and –4 **(g)** –5 and 3 **(h)** 4 (twice)

(i) 5 and 2 **(j)** 3 and 9 **(k)** 9 and –2 **(l)** 4 and 6

(m) –4 and 2 **(n)** –2 and 3 **(o)** –3 and 10 **(p)** –7 and 2

(q) –3 and 5 **(r)** –6 and 2

5. **(a)** $-\frac{5}{2}$ and –1 **(b)** $-\frac{1}{2}$ and –5 **(c)** $-\frac{1}{3}$ and –3 **(d)** $-\frac{1}{3}$ and –2

(e) $-\frac{5}{3}$ and –1 **(f)** $\frac{3}{5}$ and –2 **(g)** $\frac{1}{2}$ and –3 **(h)** $\frac{2}{3}$ and 1

(i) $\frac{1}{5}$ and 3 **(j)** $\frac{2}{3}$ and 4 **(k)** $\frac{7}{2}$ and –1 **(l)** $\frac{1}{6}$ and –1

(m) $-\frac{1}{3}$ and 1 **(n)** $\frac{3}{4}$ and –2 **(o)** $-\frac{1}{2}$ and $\frac{3}{2}$ **(p)** $-\frac{5}{3}$ and 1

(q) $\frac{2}{9}$ and $-\frac{1}{4}$ **(r)** $-\frac{1}{7}$ and 4

USING QUADRATIC FORMULA

1. **(a)** $-\frac{1}{3}$ and –2 **(b)** $-\frac{1}{2}$ and –2 **(c)** $-\frac{5}{3}$ and –1 **(d)** $-\frac{9}{2}$ and –1

(e) $-\frac{1}{2}$ and –5 **(f)** $-\frac{2}{3}$ and –3 **(g)** $\frac{1}{2}$ and 3 **(h)** $\frac{3}{2}$ and 1

(i) $\frac{2}{5}$ and 3 **(j)** $\frac{2}{5}$ and 1 **(k)** $\frac{2}{3}$ and $\frac{1}{2}$ **(l)** $\frac{3}{4}$ and 2

(m) $-\frac{1}{3}$ and 1 **(n)** $\frac{3}{2}$ and –1 **(o)** $-\frac{1}{2}$ and $\frac{3}{2}$ **(p)** $\frac{1}{2}$ and –4

(q) $-\frac{1}{6}$ and 2 **(r)** $\frac{2}{3}$ or –4

2. **(a)** –1·38 and –3·62 **(b)** –0·23 and –8·77 **(c)** –0·27 and –3·73

(d) –0·59 and –3·41 **(e)** –0·46 and –6·54 **(f)** –0·68 and –7·32

(g) 4·79 and 0·21 **(h)** 11·66 and 0·34 **(i)** 5·65 and 0·35

(j) 9·12 and 0·88 **(k)** 2·62 and 0·38 **(l)** 6·37 and 0·63

(m) 0·36 and –8·36 **(n)** 1·16 and –5·16 **(o)** 2·16 and –4·16

3. **(a)** –1 and –1·67 **(b)** –0·36 and –4·14 **(c)** –0·22 and –2·28

(d) –0·15 and 1·65 **(e)** –0·57 and –1·77 **(f)** –0·26 and –1·54

(g) 0·70 and 0·18 **(h)** –3·23 and 0·23 **(i)** 1·58 and 0·42

(j) –1·45 and –0·55 **(k)** 1·24 and 0·16 **(l)** –0·35 and 4·35

(m) 0·22 and –1·82 **(n)** –1·09 and 0·76 **(o)** 0·23 and –0·43

(p) –2·93 and 0·68 **(q)** –0·47 and 0·90 **(r)** 1·68 and –2·68

4. **(a)** –0·697 and –4·30 **(b)** –0·382 and –2·62 **(c)** –0·258 and –7·74

(d) –1·21 and –5·79 **(e)** –0·354 and –5·65 **(f)** –0·551 and –5·45

(g) 0·438 and 4·56 **(h)** 0·469 and 8·53 **(i)** 0·807 and 6·19

(j) 0·310 and 9·69 **(k)** 1·17 and 6·83 **(l)** 0·586 and 3·41

(m) –13·5 and 1·48 **(n)** –12·2 and 1·23 **(o)** –9·75 and 1·75

(p) –4·85 and 1·85 **(q)** –4·46 and 2·46 **(r)** –5·28 and 2·28

(s) –0·146 and –0·854 **(t)** –1·68 and 1·08 **(u)** –0·312 and 4·81

(v) 0·631 and 0·227 **(w)** 1·21 and –2·21 **(x)** 1·92 and –1·17

MORE QUADRATICS

1. **(a)** –3 and 1; $x = -1$; (–1, –4) Minimum; (0, –3)

(b) –2 and 4 ; $x = 1$; (1, –9) Minimum; (0, –8)

(c) –1 and 5; $x = 2$; (1, –9) Minimum; (0, –5)

(d) –6 and 0; $x = -3$; (–3, –9) Minimum; (0, 0)

(e) 0 and 4; $x = 2$; (2, –4) Minimum; (0, 0)

(f) 0 and 8; $x = 4$; (4, 16) Maximum; (0, 0)

(g) –4 and 2; $x = -1$; (–1, 9) Maximum; (0, 8)

(h) –1 and 7; $x = 3$; (3, 16) Maximum; (0, 7)

(i) 3 and 7; $x = 5$; (5, –4) Minimum; (0, 21)

(j) –1 and 4; $x = 1{\cdot}5$; (1·5, –6·25) Minimum; (0, –4)

(k) –6 and –1; $x = -3{\cdot}5$; (–3·5, –6·25) Minimum; (0, 6)

(l) 0 and 5; $x = 2{\cdot}5$; (2·5, 6·25) Maximum; (0, 0)

(m) 2 and **–**5; $x = -1{\cdot}5$; (**–**1·5, 12·25) Maximum; (0, 10)

(n) –4 and 4; $x = 0$; (0, 16) Maximum; (0, 16)

(o) –3 and 3; $x = 0$; (0, –9) Minimum; (0, –9)

2. **(a)** A(–2, 0); B(8, 0); C(3, –25); D(0, –16)

(b) E(–11, 0); F(3, 0); G(–4, –49); H(0, –33)

(c) I(–6, 0); J(–2, 0); K(–4, –4); L(0, 12)

(d) M(–6, 0); N(4, 0); O(–1, –25); P(0, –24)

(e) R(–1, 0); S(5, 0); T(2, –9); U(0, –5)

(f) A(1, 0); B(7, 0); C(4, –9); D(0, 7)

PROBLEMS INVOLVING QUADRATIC EQUATIONS

1. **(a)** $x^2 + (x+7)^2 = 13^2$; $x = 5$ **(b)** 30 cm **2.** $n = 9$; 54 cm²

3. $n = 10$; 65 mm² **4.** **(a)** $80 - x$ **(b)** $x(80 - x) = 1500$; 30×50

5. **(a)** $130 - x$ **(b)** $x(130 - x) = 4000$; 80m

WORKING with QUADRATICS

EXAM QUESTIONS

1. **(a)** (¾ , ½) **(b)** minimum **(c)** $x = ¾$ **2.** $(-3, -5)$

3. 0·9 and – 1·2 **4**. **(a)** $y = (x + 2)(x - 6)$ **(b)** (2, – 16); minimum

(c) $y = (x - 2)^2 - 16$.

5. 0·6 and –3·9 **6.** (–½,–¾) ; minimum

7. 3·8 and –1·3 **8**. $y = (x - 1)^2 + 6$. **9.** 0·65 and –6·2

10. **(a)** (–1, –2) minimum **(b)** $x = -1$ **(c)** $y = (x + 1)^2 - 2$

11. 0·4 and –3·9 **12.** **(a)** (¾, ⅚) minimum **(b)** $x = ¾$

13. 5·67 and –1·67

14. **(a)** $y = (x + 2)(x - 4)$ **(b)** (0, –8) **(c)** (1, –9); min **(d)** $x = 1$

15. **(a)** (4, 7); minimum **(b)** B **16.** 2·8 and 0·2

17. **(a)** (3, 9) **(b)** $x = 3$ **(c)** (6, 0)

18. **(a)** (3, –1); minimum **(b)** $x = 3$ **(c)** (4, 0) and (2, 0)

19. **(a)** (–2, 9) maximum **(b)** $x = -2$

20. **(a)** $y = (x-3)^2 - 9$ **(b)** 6 units

21. **(a)** $y = (x-2)^2 - 1$. **(b)** $x = 2$ **(c)** (0, 3) **(d)** A(1, 0)

22. **(a)** proof **(b)** $x = 4$

DISCRIMINANT

1. **(a)** 4 **(b)** 0 **(c)** 36 **(d)** 49 **(e)** 9 **(f)** 0

(g) 1 **(h)** –23 **(i)** 196 **(j)** 169 **(k)** 28 **(l)** –80

(m) 17 **(n)** –55 **(o)** 16 **(p)** 0 **(q)** –11 **(r)** 0

2. **(a)** real, rational, distinct **(b)** equal **(c)** real, rational, distinct

(d) real, rational, distinct **(e)** real, rational, distinct **(f)** equal

(g) real, rational, distinct **(h)** non real **(i)** real, rational, distinct

(j) real, rational, distinct **(k)** real, irrational, distinct **(l)** non real

(m) real, irrational, distinct **(n)** non real **(o)** Real, rational, distinct

(p) equal **(q)** non real **(r)** equal

3. **(a)** $b^2 - 4ac > 0$ **(b)** $b^2 - 4ac < 0$ **(c)** $b^2 - 4ac = 0$ **(d)** $b^2 - 4ac > 0$

(e) $b^2 - 4ac = 0$ **(f)** $b^2 - 4ac < 0$ **(g)** $b^2 - 4ac > 0$ **(h)** $b^2 - 4ac > 0$

(i) $b^2 - 4ac > 0$ **(j)** $b^2 - 4ac < 0$ **(k)** $b^2 - 4ac < 0$ **(l)** $b^2 - 4ac > 0$

4. **(a)** –4 **(b)** 12·5 **(c)** 0·2

(d) 0·5 or 1·5 **(e)** $5\frac{1}{3}$ **(f)** $-2\frac{9}{20}$

5. **(a)** 4 **(b)** $\frac{1}{2}$ **(c)** $\pm\sqrt{5}$

(d) 9 **(e)** $\frac{5}{8}$ **(f)** ± 4

EXAM QUESTIONS

1. 41, real, irrational and distinct **2.** **(a)** 13 **(b)** real, irrational and distinct

3. 1 **4.** Non real **5.** 13, real, irrational and distinct

3.1 APPLYING the THEOREM of PYTHAGORAS

1. **(a)** 9·43 **(b)** 21·3 **(c)** 13 **(d)** 10·2 **(e)** 1·05 **(f)** 5·07

(g) 12·4 **(h)** 26·9 **(i)** 2·4

2. **(a)** 10 **(b)** 15·6 **(c)** 180cm²

3. **(a)** 21 **(b)** 357cm²

4. **(a)** 7·07 **(b)** 12·5 **(c)** 8·06 **(d)** 10·2

5. **(a)** 11.7cm **(b)** 12·7cm

6. **(a)** 20cm **(b)** 15·0cm

7. 19·2km **8.** 427·2km **9.** 9·46km

10. 7·9km **11.** 15·6mm **12.** 16·6cm

13. 4·6m **14.** 4·4m **15.** 16cm

16. 6·6cm **17.** 16·9m **18.** 1·92m

19. **(i)** 2·9cm **(ii)** 25·4cm² **(iii)** 7·35cm³

20. proofs

21. **(a)** no **(b)** no **(c)** no

(d) yes **(e)** no **(f)** yes

EXAM QUESTIONS – PYTHAGORAS

1. Mechanism will work since 7·1 > 7 **2.** 34·8cm

3. 67cm **4.** Suitable since 11·3 > 11 **5.** 41·2cm

6. £952 **7.** 21·5m **8.** 3·3m **9.** 37cm

10. 20·7m **11.** **(a)** AB = 8mm; BC = 4mm **(b)** 6·9mm

12. 7·7cm **13.** 5·06m or 506 cm **14.** 15cm **15.** 33cm

16. **(a)** 15cm **(b)** 16cm **(c)** proof **(d)** 30cm; 225cm²

17. supports are vertical **18.** 13·9 > 13·89 so just long enough

3.2 APPLYING PROPERTIES OF SHAPES

1. **(a)** square **(b)** 0; rotational symmetry **(c)** equal

(d) diagonals; bisect; right **(e)** trapezium

2. 1925cm² **3.** **(a)** 16 100cm² **(b)** 1·61m³

4. **(a)** 24cm² **(b)** 432cm² **(c)** 50%

5. **(a)** pentagon; 72°; 108°, 72° **(b)** octagon; 45°; 135°; 45°

(c) hexagon; 60°; 120°; 60°

6. **(a)** right angled scalene **(b)** acute angled isosceles

(c) obtuse angled scalene

7. Acute angled isosceles; 23·8 units²

8. 30°, 60°, 90° and 32°, 36°, 112° angles of a triangle add up to 180°

9. **(a)** 37° **(b)** 62° and 56° **(c)** 40°

10. 5760cm² **11.** **(a)** £1.50 **(b)** £3

12. £792.18 **13.** **(a)** false **(b)** true **(c)** false **(d)** true

14. **(a)** 47·1m **(b)** 56·52cm **15.** 43·96cm

16. 33·3cm **17.** 596·6mm; £2.75 **18.** 9m **19.** 12·1cm

20. **(a)** 452·16cm² **(b)** 706·5cm² **21.** 1017·36cm² **22.** 192mm²

23. **(a)** 12·56cm² **(b)** 188·4cm² **(c)** 240cm² **(d)** 21·5%

24. **(a)** 14·13m² **(b)** 45·87m² **(c)** £364.93

25. 2·5cm; 15·7cm

3.2 RELATIONSHIP between the CENTRE, CHORD and PERPENDICULAR BISECTOR

1. **(a)** 90° **(b)** 45° **(c)** 90° **(d)** 55° **(e)** 90°
(f) 43° **(g)** 90° **(h)** 18° **(i)** 90° **(j)** 63°
(k) 90° **(l)** 78°

2. **(a)** 9·9 cm **(b)** 8·5 cm **(c)** 6·4 cm **(d)** 9·2 cm

3. **(a)** 40^o **(b)** 40^o **(c)** 50^o **(d)** 33^o **(e)** 33^o
(f) 57^o **(g)** 28^o **(h)** 62^o **(i)** 62^o **(j)** 118^o
(k) 118^o **(l)** 31^o **(m)** 31^o **(n)** 31^o **(o)** 31^o

4. **(a)** 4·5 cm **(b)** 5·7 cm **(c)** 7·2 cm **(d)** 3 cm **(e)** 8 cm
(f) 9·2 cm

5. **(a)** $36{\cdot}9^o$ **(b)** 24·1 cm **(c)** 9·0 cm **(d)** 12·6 cm
(e) 23·7 cm **(f)** 8 cm **6.** 37·6 cm

TANGENTS and ANGLES

1. **(a)** 90^o **(b)** 20^o **(c)** 110^o **(d)** 90^o **(e)** 60^o
(f) 30^o **(g)** 35^o **(h)** 35^o **(k)** 90^o **(m)** 65^o
(n) 90^o **(p)** 55^o **(q)** 90^o **(r)** 45^o

2. **(a)** 6 cm **(b)** 13 cm **(c)** 24 cm

3. **(a)** $33{\cdot}7^o$ **(b)** 10·4 cm **(c)** 14·3 cm

EXAM QUESTIONS

1. **(a)** 50cm **(b)** 14cm **2.** **(a)** 100cm **(b)** 171cm

3. 20cm **4.** 102cm **5.** 54^o **6.** 9·6cm **7.** 23^o **8.** 132^o

9. 28^o **10.** **(a)** 112^o **(b)** 60·6cm **11.** 36cm **12.** 71·4cm

13. 76^o **14.** 8cm **15.** 11·3cm

3.3 USING SIMILARITY

LENGTH

1. **(a)** **(i)** s.f. = $\frac{3}{2}$ or 1·5 **(ii)** 13·5cm **(b)** **(i)** s.f. = $\frac{2}{3}$ of 0·66... **(ii)** 20cm

(c) **(i)** s.f. = $\frac{5}{2}$ of 2·5 **(ii)** 45cm **(d)** **(i)** s.f. = $\frac{3}{5}$ or 0·6 **(ii)** 168mm

2. **(a)** $x = 30$mm **(b)** $x = 32{\cdot}5$cm

3. **(a)** Because they are equiangular **(b)** CD = 18cm **(c)** 81 cm^2

4. **(a)** $x = 13 \cdot 5\,\text{cm}$ **(b)** $x = 14 \cdot 4\,\text{m}$ **5.** ST = 16cm **6.** distance = $0 \cdot 7\,\text{m}$

SIMILARITY and AREA

1. **(a)** s.f.(L) = 2; s.f. (A) = 4; $A = 64\text{cm}^2$

(b) s.f.(L) = 3; s.f. (A) = 9; $A = 864\text{mm}^2$

(c) s.f.(L) = 1·5; s.f. (A) = 2·25; $A = 90\text{mm}^2$

(d) s.f.(L) = 2·4; s.f. (A) = 5·76; $A = 288\text{cm}^2$

(e) s.f.(L) = 4; s.f. (A) = 16; $A = 352\text{cm}^2$

(f) s.f.(L) = 1·8; s.f. (A) = 3·24; $A = 388 \cdot 8\text{cm}^2$

2. **(a)** s.f.(L) = 0·5; s.f. (A) = 0·25; $A = 17 \cdot 5\text{cm}^2$

(b) s.f.(L) = 0·25; s.f. (A) = 0·0625; $A = 288\text{mm}^2$

(c) s.f.(L) = 0·8; s.f. (A) = 0·64; $A = 96\text{cm}^2$

(d) s.f.(L) = 0·75; s.f. (A) = 0·5625; $A = 225\text{mm}^2$

3. **(a)** 88 cm^2 **(b)** 166 mm^2 **(c)** 49cm^2 **(d)** 72 mm^2

SIMILARITY and VOLUME

1. **(a)** s.f.(L) = 2; s.f.(V) = 8; $V = 384\text{cm}^3$

(b) s.f.(L) = 3; s.f.(V) = 27; $V = 5832\text{mm}^3$

(c) s.f.(L) = 1·5; s.f.(V) = 3·375; $V = 243\text{mm}^3$

(d) s.f.(L) = 2·4; s.f.(V) = 13·824; $V = 276 \cdot 48\text{cm}^3$

(e) s.f.(L) = 4; s.f.(V) = 64; $V = 576\text{cm}^3$

(f) s.f.(L) = 1·4; s.f.(V) = 2·744; V = $1097 \cdot 6\text{cm}^3$

2. **(a)** s.f.(L) = 0·5; s.f.(V) = 0·125; $V = 46\text{cm}^3$

(b) s.f.(L) = 0·25; s.f.(V) = 0·015625; $V = 2 \cdot 25\text{mm}^3$

(c) s.f.(L) = 0·75; s.f.(V) = 0·421875; $V = 384 \cdot 75\text{cm}^3$

(d) s.f.(L) = 0·6; s.f.(V) = 0·216; $V = 49 \cdot 68\text{mm}^3$

3. **(a)** 1200 ml **(b)** 270 ml **(c)** 6·75 litres

SIMILARITY – EXAM QUESTIONS

1. no, will burn for 4 times the time **2.** priced correctly

3. rug is too small since 69·1 < 72 **4.** 7cm

4.1 WORKING with TRIGONOMETRIC FUNCTIONS – BASIC GRAPHS

1. Sine graph drawn **2.** Cosine graph drawn **3.** Tangent graph drawn

4.1 WORKING with TRIGONOMETRIC FUNCTIONS (1)

1. **(a)** $y = 5 \sin x^o$ **(b)** $y = 3 \sin x^o$ **(c)** $y = 0{\cdot}5 \sin x^o$ **(d)** $y = 4 \cos x^o$

(e) $y = 12 \cos x^o$ **(f)** $y = \frac{1}{4} \cos x^o$ **(g)** $y = 1{\cdot}5 \sin x^o$ **(h)** $y = 2{\cdot}7 \cos x^o$

(i) $y = 3{\cdot}3 \sin x^o$ **(j)** $y = -8 \cos x^o$ **(k)** $y = -6 \sin x^o$ **(l)** $y = -20 \cos x^o$

(m) $y = -2{\cdot}8 \sin x^o$ **(n)** $y = \frac{3}{4} \sin x^o$ **(o)** $y = 0{\cdot}6 \cos x^o$

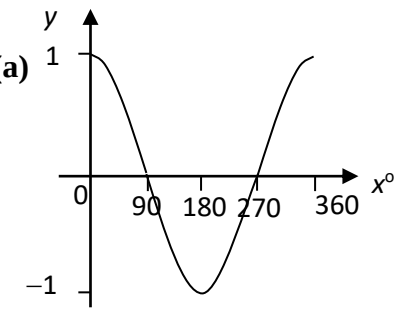
2. **(a)** $y = 3 \sin 2x^o$ **(b)** $y = 5 \sin 3x^o$ **(c)** $y = 2 \cos 4x^o$ **(d)** $y = 10 \cos 2x^o$

(e) $y = 7 \sin 2x^o$ **(f)** $y = 4 \cos 3x^o$ **(g)** $y = -6 \sin 3x^o$ **(h)** $y = -5 \cos 2x^o$

(i) $y = 3 \sin \frac{1}{2} x^o$ **(j)** $y = 9 \cos \frac{1}{2} x^o$ **(k)** $y = 20\cos^1/_3 x^o$ **(l)** $y = 3 \cos {}^3/_2 x^o$

(m) $y = \tan x^o$ **(n)** $y = \tan 2x^o$ **(o)** $y = \tan \frac{1}{2} x^o$ **(p)** $y = \tan 4x^o$

3. **(a)**



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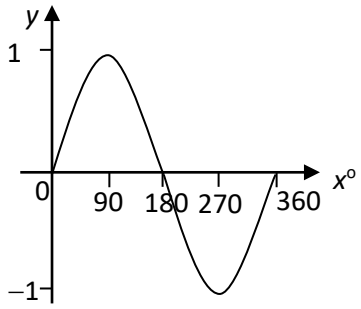

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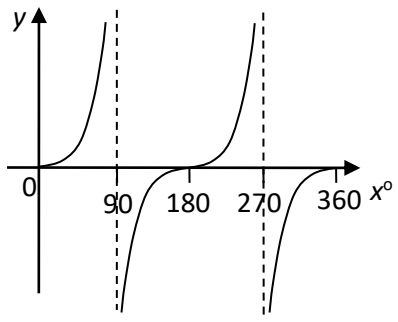

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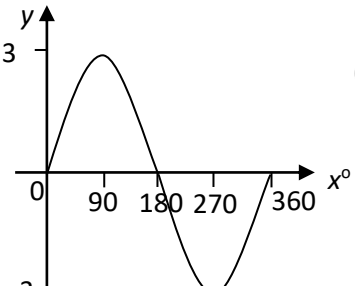

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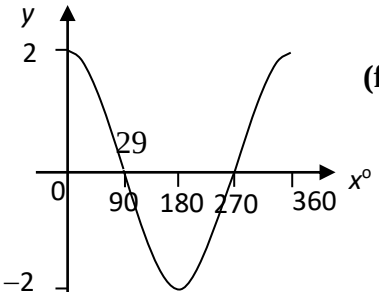

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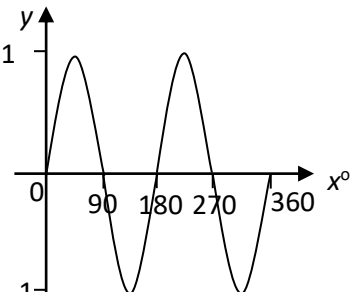

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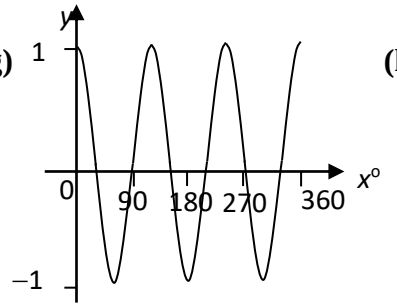

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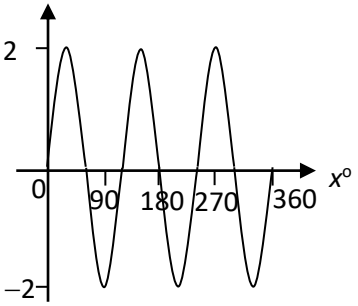

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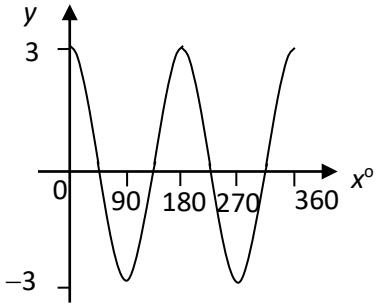

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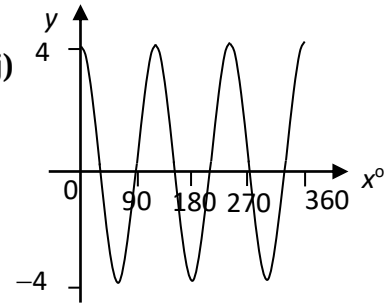

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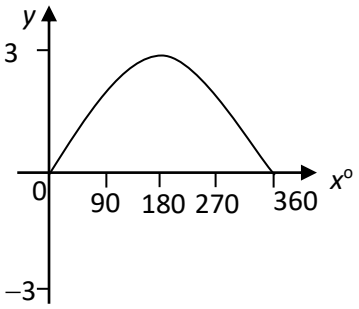

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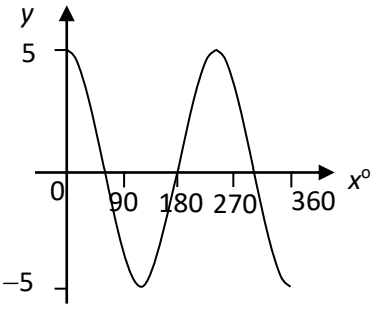

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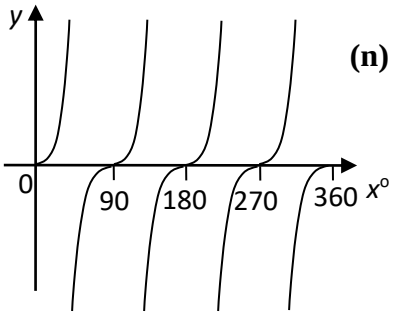

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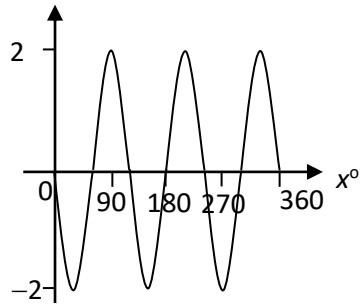

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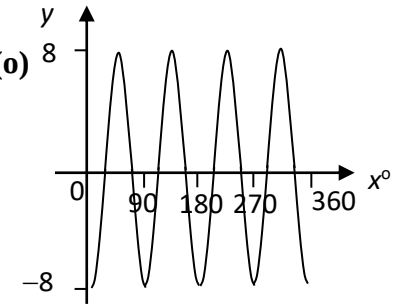


4. **(a)** $y = \sin (x + 10)^o$ **(b)** $y = \sin (x - 40)^o$ **(c)** $y = \cos (x - 25)^o$

(d) $y = \cos (x + 30)^o$ **(e)** $y = \sin (x + 15)^o$ **(f)** $y = \cos (x - 30)^o$

(g) $y = \cos (x + 45)^o$ **(h)** $y = \sin (x - 37)^o$ **(i)** $y = \sin (x - 23)^o$

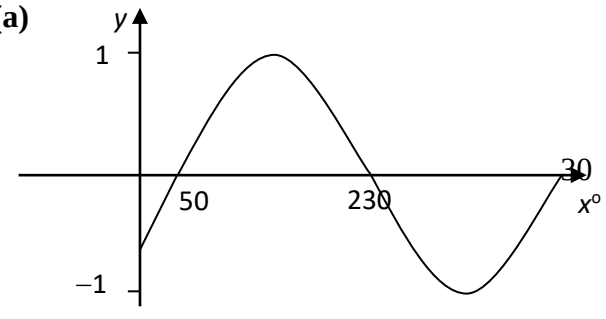
(j) $y = \cos (x - 18)^o$

5. **(a)** $y = \tan (x - 45)^o$ **(b)** $y = 3 \sin (x - 30)^o$ **(c)** $y = 5 \cos (x + 35)^o$

(d) $y = 2 \sin (x + 25)^o$ **(e)** $y = 6 \sin (x + 8)^o$ **(f)** $y = 1.5 \cos (x - 25)^o$

(g) $y = 5 \cos (x - 20)^o$ **(h)** $y = 4 \sin (x + 75)^o$

6. **(a)**



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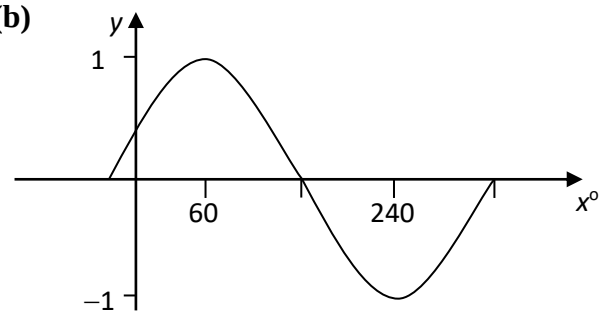

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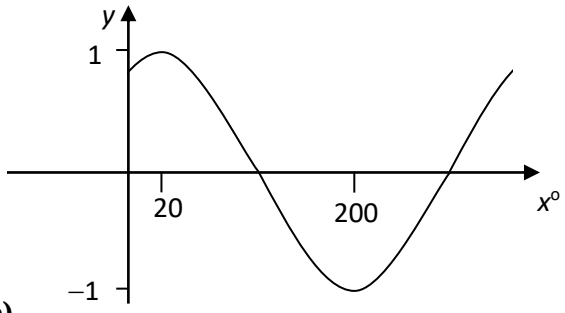

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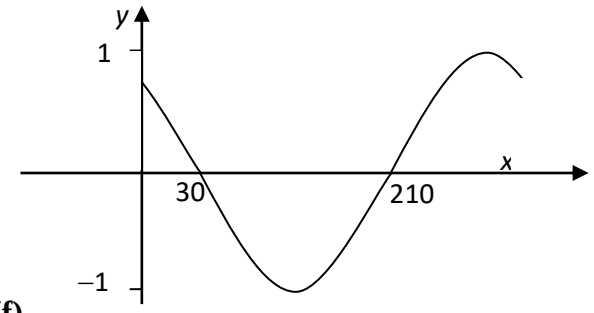

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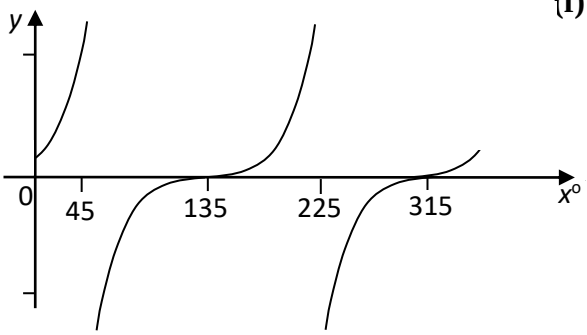

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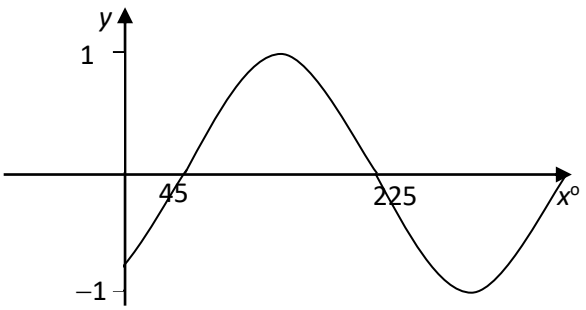


4.1 WORKING with TRIGONOMETRIC FUNCTIONS (2)

1. **(a)** 5; $y = 5\sin x^o + 2$ **(b)** 3; $y = 3\sin x^o - 1$ **(c)** 0·5; $y = 0{\cdot}5\sin x^o - 0{\cdot}5$

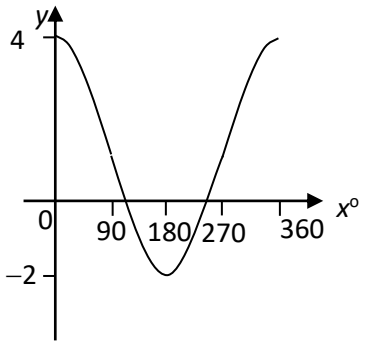
(d) 4; $y = 4\cos x^o + 1$ **(e)** 12; $y = 12\cos x^o - 3$ **(f)** 4; $y = 4\cos 2x^o - 1$

(g) 1·5; $y = 1{\cdot}5\sin x^o - 0{\cdot}5$ **(h)** 2·7; $y = 2{\cdot}7\cos 3x^o - 0{\cdot}2$

(i) 3·3; $y = 3{\cdot}3\sin 2x^o - 0{\cdot}8$ **(j)** 8; $y = -8\cos x^o + 4$

(k) 6; $y = -6\sin 3x^o + 3$ **(l)** 20; $y = -20\cos x^o - 5$

2. **(a)**



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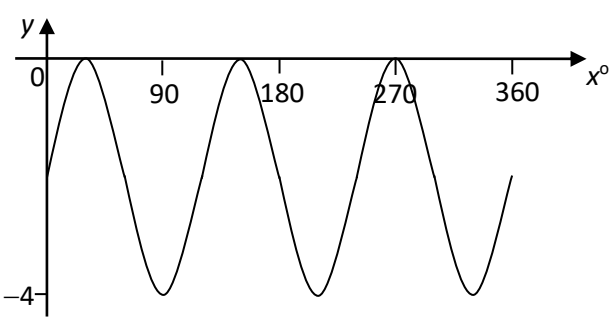

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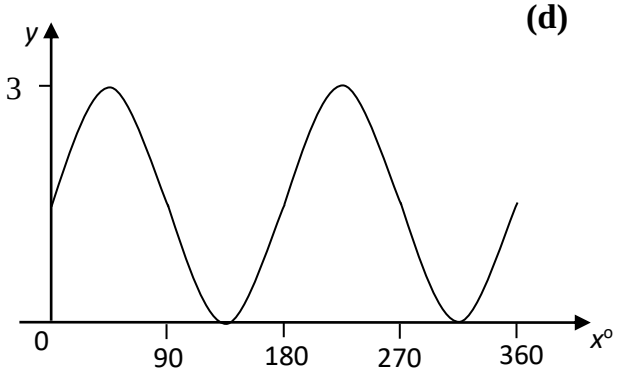
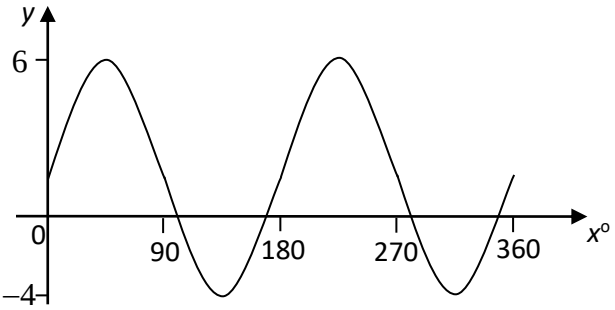
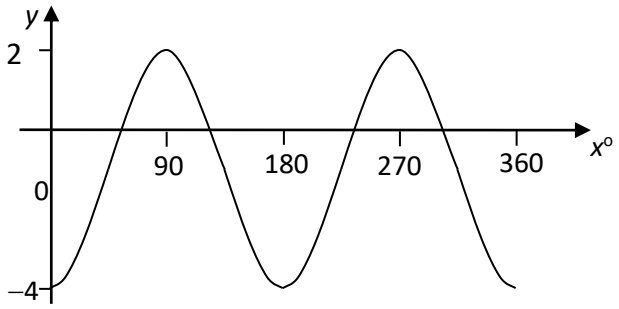


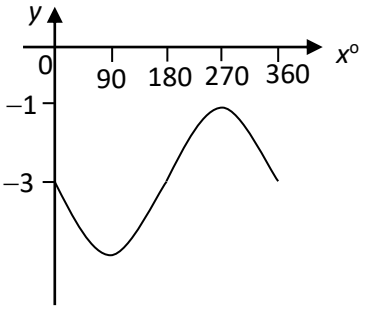
(d)



(e)



(f)



4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES

Sine, cosine and tangent of angles 0 – 360°

1.

	$0 < x < 90$	$90 < x < 180$	$180 < x < 270$	$270 < x < 360$
$\sin x^o$	+	+	–	–
$\cos x^o$	+	–	–	+
$\tan x^o$	+	–	+	–

2. **(a)** + **(b)** – **(c)** – **(d)** + **(e)** + **(f)** +

(g) + **(h)** + **(i)** – **(j)** – **(k)** – **(l)** –

3. **(a)** $\cos 22^o$ **(b)** $-\tan 57^o$ **(c)** $-\sin 45^o$ **(d)** $\sin 15^o$

(e) $\tan 16^o$ **(f)** $-\cos 5^o$ **(g)** $\tan 66^o$ **(h)** $\sin 48^o$

(i) $-\cos 10^o$ **(j)** $-\sin 38^o$ **(k)** $-\cos 20^o$ **(l)** $-\tan 50^o$

4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES

EXACT VALUES

1. **(a)** $\frac{1}{2}$ **(b)** $\frac{1}{2}$ **(c)** $-\frac{1}{2}$ **(d)** $-\frac{1}{2}$

(e) $\frac{\sqrt{3}}{2}$ **(f)** $-\frac{\sqrt{3}}{2}$ **(g)** $-\frac{\sqrt{3}}{2}$ **(h)** $\frac{\sqrt{3}}{2}$

(i) $\frac{1}{\sqrt{3}}$ **(j)** $-\frac{1}{\sqrt{3}}$ **(k)** $\frac{1}{\sqrt{3}}$ **(l)** $-\frac{1}{\sqrt{3}}$

2. **(a)** $\frac{\sqrt{3}}{2}$ **(b)** $\frac{\sqrt{3}}{2}$ **(c)** $-\frac{\sqrt{3}}{2}$ **(d)** $-\frac{\sqrt{3}}{2}$

(e) $\frac{1}{2}$ **(f)** $-\frac{1}{2}$ **(g)** $-\frac{1}{2}$ **(h)** $\frac{1}{2}$

(i) $\sqrt{3}$ **(j)** $-\sqrt{3}$ **(k)** $\sqrt{3}$ **(l)** $-\sqrt{3}$

3. **(a)** $\frac{1}{\sqrt{2}}$ **(b)** $\frac{1}{\sqrt{2}}$ **(c)** $-\frac{1}{\sqrt{2}}$ **(d)** $-\frac{1}{\sqrt{2}}$

(e) $\frac{1}{\sqrt{2}}$ **(f)** $-\frac{1}{\sqrt{2}}$ **(g)** $-\frac{1}{\sqrt{2}}$ **(h)** $\frac{1}{\sqrt{2}}$

(i) 1 **(j)** -1 **(k)** 1 **(l)** -1

4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES

PERIOD

1. **(a)** 12 **(b)** 40 **(c)** 120^o **(d)** 90^o

(e) 90^o **(f)** 180^o **(g)** 360^o **(h)** 180^o

(i) 240^o **(j)** 720^o

2.	**(a)**	180^o	**(b)**	90^o	**(c)**	180^o	**(d)**	60^o
	(e)	90^o	**(f)**	120^o	**(g)**	240^o	**(h)**	80^o
	(i)	360^o	**(j)**	45^o	**(k)**	30^o	**(l)**	30^o
	(m)	10^o	**(n)**	40^o	**(o)**	12^o	**(p)**	24^o
	(q)	36^o	**(r)**	45^o				

3.	**(a)**	720^o	**(b)**	540^o	**(c)**	1440^o	**(d)**	900^o
	(e)	2160^o	**(f)**	540^o	**(g)**	180^o	**(h)**	120^o
	(i)	90^o	**(j)**	180^o	**(k)**	180^o	**(l)**	90^o

SOLVING BASIC EQUATIONS

1.	**(a)**	30^o, 150^o	**(b)**	30^o, 330^o	**(c)**	45^o, 225^o
	(d)	120^o, 240^o	**(e)**	150^o, 330^o	**(f)**	240^o, 300^o
	(g)	60^o, 240^o	**(h)**	45^o, 135^o	**(i)**	45^o, 315^o
	(j)	225^o, 315^o	**(k)**	150^o, 210^o	**(l)**	120^o, 300^o

2.	**(a)**	$18{\cdot}2^o$, $161{\cdot}8^o$	**(b)**	$64{\cdot}8^o$, $295{\cdot}2^o$	**(c)**	79^o, 259^o
	(d)	95^o, 265^o	**(e)**	139^o, 319^o	**(f)**	191^o, 349^o
	(g)	85^o, 265^o	**(h)**	44^o, 136^o	**(i)**	41^o, 319^o
	(j)	201^o, 339^o	**(k)**	133^o, 227^o	**(l)**	165^o, 345^o

3.	**(a)**	30^o, 150^o	**(b)**	$48{\cdot}2^o$, $311{\cdot}8^o$	**(c)**	59^o, 239^o
	(d)	120^o, 240^o	**(e)**	104^o, 284^o	**(f)**	$228{\cdot}6^o$, $311{\cdot}4^o$
	(g)	78^o, 258^o	**(h)**	$23{\cdot}6^o$, $156{\cdot}4^o$	**(i)**	$80{\cdot}4^o$, $279{\cdot}6^o$
	(j)	202^o, 338^o	**(k)**	$144{\cdot}9^o$, $215{\cdot}1^o$	**(l)**	138^o, 318^o

4.	**(a)**	90^o	**(b)**	180^o	**(c)**	45^o, 225^o
	(d)	210^o, 330^o	**(e)**	60^o, 300^o	**(f)**	$26{\cdot}6^o$, $206{\cdot}6^o$
	(g)	$41{\cdot}4^o$, $318{\cdot}6^o$	**(h)**	$41{\cdot}8^o$, $138{\cdot}2^o$	**(i)**	$113{\cdot}6^o$, $246{\cdot}4^o$
	(j)	$33{\cdot}7^o$, $213{\cdot}7^o$	**(k)**	$109{\cdot}5^o$, $250{\cdot}5^o$	**(l)**	$205{\cdot}4^o$, $334{\cdot}6^o$

5.	**(a)**	$104{\cdot}5^o$, $255{\cdot}5^o$	**(b)**	$44{\cdot}4^o$, $135{\cdot}6^o$	**(c)**	$84{\cdot}3^o$, $264{\cdot}3^o$
	(d)	60^o, 300^o	**(e)**	45^o, 225^o	**(f)**	$126{\cdot}9^o$, $233{\cdot}1^o$
	(g)	$23{\cdot}6^o$, $156{\cdot}4^o$	**(h)**	120^o, 240^o	**(i)**	$53{\cdot}1^o$, $126{\cdot}9^o$
	(j)	$118{\cdot}1^o$, $241{\cdot}9^o$	**(k)**	$45{\cdot}6^o$, $134{\cdot}4^o$	**(l)**	$33{\cdot}2^o$, $146{\cdot}8^o$

IDENTITIES INVOLVING $\cos^2 x + \sin^2 x = 1$ and $\tan x = \sin x / \cos x$

1. **(a)** $3(\cos^2 x + \sin^2 x) = 3 \times 1 = 3$ **(b)** $\sin^2 x$ **(c)** $\cancel{\cos A} \times \frac{\sin A}{\cancel{\cos A}} = \sin A$

(d) $5(1 - \sin^2 B) = 5\cos^2 B$ **(e)** $\frac{4\sin a^o}{4\cos a^o} = \frac{\sin a^\circ}{\cos a^\circ} = \tan a^\circ$

(f) $\frac{4\tan x^o}{2\cos x^o} = \frac{2\tan x^\circ}{\cos x^\circ} = \frac{2\frac{\sin x^\circ}{\cos x^\circ}}{\cos x} = 2\frac{\sin x^\circ}{\cos x^\circ} \times \frac{1}{\cos x^\circ} = \frac{2\sin x^\circ}{\cos^2 x}$

(g) $\frac{(1-\sin^2 x)}{2\cos x} = \frac{\cos^2 x}{2\cos x} = \frac{\cos x}{2}$ **(h)** $\frac{8-8\cos^2 x}{2\sin x} = \frac{8(1-\cos^2 x)}{2\sin x} = \frac{8\sin^2 x}{2\sin x} = 4\sin x$

(i) $\frac{3\sin x\cos x}{6\tan x} = \frac{3\sin x\cos x}{6\frac{\sin x}{\cos x}} = 3\sin x\cos x \times \frac{\cos x}{6\sin x} = \frac{\cos^2 x}{2}$

(j) $4\sin^2 A + 3\cos^2 A - 3 = 3(\sin^2 A + \cos^2 A) + \sin^2 A - 3 = 3 + \sin^2 A - 3 = \sin^2 A$

(k) $4\cos^2 B - 2\sin^2 B + 2 = 4\cos^2 B - 2(1-\cos^2 B) + 2 = 4\cos^2 B - 2 + \cos^2 B - 2 = 6\cos^2 B$

(l) $(\cos x + \sin x)^2 - 2\sin x\cos x$
$= \cos^2 x + 2\cos x\sin x + \sin^2 x - 2\sin x\cos x = \cos^2 x + \sin^2 x = 1$

(m) $\tan^2 a(1 - \sin^2 a) = \frac{\sin^2 \alpha}{\cos^2 \alpha} \times \cos^2 \alpha = \sin^2 \alpha$

2. **(a)** $3(\cos^2 A + \sin^2 A) - \cos^2 A = 3(1) - \cos^2 A = 3 - \cos^2 A$

(b) $\frac{1}{\tan x} + \tan x = \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\sin x\cos x} = \frac{1}{\sin x\cos x}$

(c) $(2\cos B + 3\sin B)^2 + (3\cos B - 2\sin B)^2$
$= 4\cos^2 B + 12\cos B\sin B + 9\sin^2 B + 9\cos^2 B - 12\cos B\sin B + 4\sin^2 B$
$= 13\cos^2 B + 13\sin^2 B = 13(\cos^2 B + \sin^2 B) = 13$

(d) $(1 + \sin x)(1 - \sin x) = 1 - \sin^2 x = \cos^2 x$

(e) $\sin\theta.\tan\theta = \sin\theta \times \frac{\sin\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta} = \frac{1-\cos^2\theta}{\cos\theta}$

3. **(a)** $\sin x^\circ = \frac{1}{2}$; $\cos x^\circ = \frac{\sqrt{3}}{2}$; $\tan x^\circ = \frac{1}{\sqrt{3}}$.

(b) $\sin^2 x^\circ + \cos^2 x^\circ = (\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2 = \frac{1}{4} + \frac{3}{4} = 1$.

(c) $\dfrac{\sin x^\circ}{\cos x^\circ} = \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{1}{2} \times \dfrac{2}{\sqrt{3}} = \dfrac{1}{\sqrt{3}} = \tan x^\circ$.

4. **(a)** $\sin a^\circ = \dfrac{2}{\sqrt{5}}$**;** $\tan a^\circ = 2$

(b) $\cos^2 a^\circ = (\dfrac{1}{\sqrt{5}})^2 = \dfrac{1}{5}; 1 - (\dfrac{2}{\sqrt{5}})^2 = 1 - \dfrac{4}{5} = \dfrac{1}{5} = \cos^2 x$

(c) $\dfrac{(\frac{2}{\sqrt{5}})^2}{(\frac{1}{\sqrt{5}})^2} = \dfrac{\frac{4}{5}}{\frac{1}{5}} = \dfrac{4}{5} \times \dfrac{5}{1} = 4;$ $\tan^2 a^\circ = 2^2 = 4$.

(d) $2[3(\dfrac{2}{\sqrt{5}}) + 4(\dfrac{1}{\sqrt{5}})] = 2(\dfrac{6}{\sqrt{5}} + \dfrac{4}{\sqrt{5}}) = \dfrac{20}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{20\sqrt{5}}{5} = 4\sqrt{5}$.

5. **(a)** $3\cos^2 a + 3\sin^2 a = 3(\cos^2 a + \sin^2 a) = 3$

(b) $(\cos x + \sin x)^2 = \cos^2 x + 2\sin x \cos x + \sin^2 x = 1 + 2\sin x \cos x$

(c) $(\cos x + \sin x)(\cos x - \sin x) = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x)$
$= \cos^2 x - 1 + \cos^2 x - 2\cos^2 x - 1$

(d) $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = \dfrac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \dfrac{1}{\cos x \sin x}$

(e) $\tan^2 p - \tan^2 p \sin^2 p = \tan^2 p(1 - \sin^2 p) = \dfrac{\sin^2 p}{\cos^2 p} \times \cos^2 p = \sin^2 p$

(f) $\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$
$= \cos^2 x - (1 - \cos^2 x) = \cos^2 x - 1 + \cos^2 x = 2\cos^2 x - 1$

(g) $3\sin^2 \theta + 2\cos^2 \theta = 2(\sin^2 \theta + \cos^2 \theta) + \sin^2 \theta = 2 + \sin^2 \theta$

(h) $\tan \alpha + \dfrac{1}{\tan \alpha} = \dfrac{\sin \alpha}{\cos \alpha} + \dfrac{\cos \alpha}{\sin \alpha} = \dfrac{\sin^2 \alpha + \cos^2 \alpha}{\sin \alpha \cos \alpha} = \dfrac{1}{\sin \alpha \cos \alpha}$

6. $(2\cos x + 5\sin x)^2 \ + \ (5\cos x - 2\sin x)^2$

$= 4\cos^2 x + 20\sin x\cos x + 25\sin^2 x + 25\cos^2 x - 20\sin x\cos x + 4\sin^2 x$

$= \ 29\cos^2 x + 29\sin^2 x = 29(\cos^2 x + \sin^2 x) = 29$

7. **(a)** $pq \ = (\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = \cos^2\theta - \sin^2\theta$

$= \cos^2\theta - (1 - \cos^2\theta) = 2\cos^2\theta \ - \ 1$

(b) $\frac{1}{2}(1 \ - \ pq) = \frac{1}{2}[1 - (2\cos^2\theta - 1)] = \frac{1}{2}(2 - 2\cos^2\theta) = 1 - \cos^2\theta = \sin^2\theta$

4.2 WORKING with TRIGONOMETRIC RELATIONSHIPS in DEGREES

EXAM QUESTIONS

1. 53·1°, 126·9° **2.** Graph drawn **3.** $\tan^2 x^\circ$

4. $a = -5$; $b = 2$ **5.** 221·8°, 318·2° **6.** $a = 3$; $b = 2$

7. 53·1°, 223·1° **8.** 48·6°, 131·4° **9.** **(a)** 4 **(b)** P(240°, –2)

10. $a = 3$; $b = 4$ **11.** Graph draw **12.** Graph drawn

13. 1 **14.** 221·8° **15.** 135·6°, 224·4°

16. any angle between 180° and 270° **17.** Graph drawn

18. 194·5°, 345·5° **19.** $1 + 2\sin x^\circ\cos x^\circ$ **20.** Proof

21. 113·6°, 246·4° **22.** C and D **23.** 104·5°, 255·5°

24. 143·1°, 323·1° **25.** $a = 2$; $b = 30$ **26.** $\cos x^\circ \sin x^\circ$

27. 107·5°, 252·5° **28.** Proof **29.** $y = 3\cos(x + 30^\circ)$